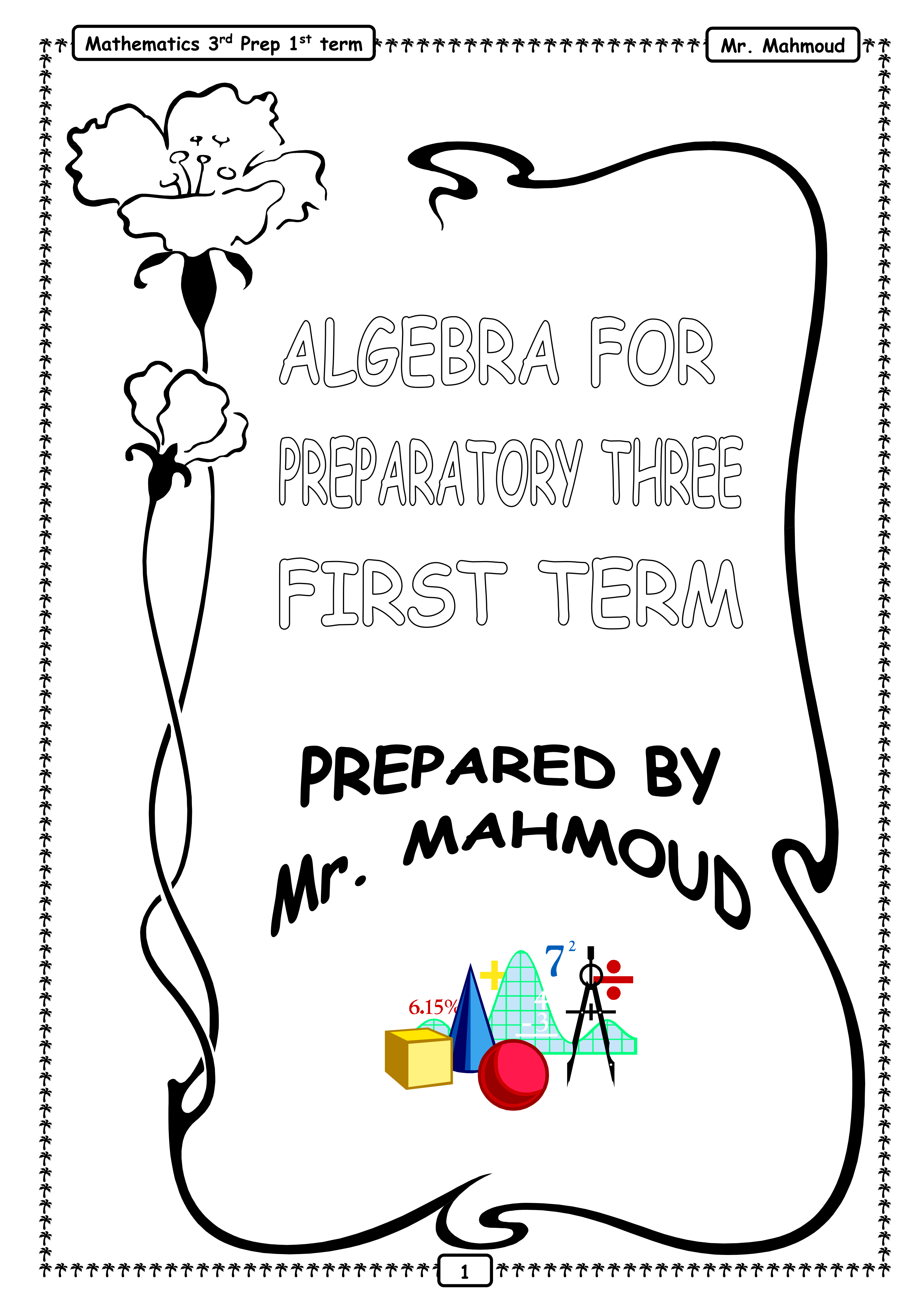


ALGEBRA FOR PREPARATORY THREE FIRST TERM

PREPARED BY

Mr. MAHMOUD

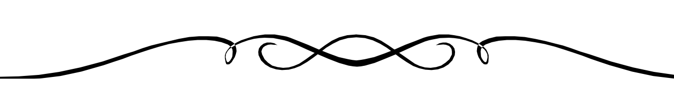
Sheet (1)
Important Revision

Compete the following with a right answer:

1	If $X + 1 = 5$, then $X =$ ....
2	If $X - 1 = 2$, then $X =$ ....
3	If $2X = 8$, then $X =$ ....
4	If $0.25X = 3$, then $X =$ ....
5	If $\frac{1}{2}X = 3$, then $X =$ ....
6	If $2X - 1 = 5$, then $X =$ ....
7	If $2X + 4 = 14$, then $X =$ ....
8	If $y^2 = 9$, then $y =$ .... or ....
9	If $5y^2 = 125$, then $y =$ .... or ....
10	If $y^2 + 4 = 20$, then $y =$ .... or ....
11	If $X^2 - 2 = 7$, then $X =$ .... or ....
12	If $5X^2 + 2 = 18$, then $X =$ .... or ....
13	If $y^3 = 1$, then $y =$ ....
14	If $3y^3 = 24$, then $y =$ ....
15	If $y^3 + 2 = 10$, then $y =$ ....
16	If $y^3 - 1 = -1$, then $y =$ ....



Homework



Compete the following with a right answer:

1	If $X + 4 = 1$, then $X =$ ....
2	If $X - 4 = 7$, then $X =$ ....
3	If $5X = 30$, then $X =$ ....
4	If $\frac{1}{4}X = 5$, then $X =$ ....
5	If $5X - 4 = 6$, then $X =$ ....
6	If $5X + 4 = 34$, then $X =$ ....
7	If $y^2 = 25$, then $y =$ .... or ....
8	If $4y^2 = 36$, then $y =$ .... or ....
9	If $y^2 + 3 = 28$, then $y =$ .... or ....
10	If $y^2 - 5 = 15$, then $y =$ .... or ....
11	If $4X^2 + 1 = 10$, then $X =$ .... or ....
12	If $y^3 = 27$, then $y =$ ....
13	If $5y^3 = 40$, then $y =$ ....
14	If $y^3 + 4 = 220$, then $y =$ ....
15	If $y^3 - 4 = 121$, then $y =$ ....

The most important mathematical notation

$\mathbb{N}$	the set of natural numbers	$\overline{X}$	mean
$\mathbb{Z}$	the set of integers	σ	standard deviation
$\mathbb{Q}$	the set of rational numbers	$\sum$	sum
$\grave{\mathbb{Q}}$	the set of irrational numbers	$\perp$	perpendicular to
$\mathbb{R}$	the set of real numbers	//	parallel
$\mathbb{R}^*$	the set of non-zero real numbers	$\equiv$	congruence
$\sqrt{a}$	square root of the number a	$\overline{AB}$	line segment $\overline{AB}$
$\sqrt[3]{a}$	cubic root of the number a	$\overrightarrow{AB}$	ray $\overrightarrow{AB}$
$[a , b]$	closed interval	$\overleftrightarrow{AB}$	straight line $\overleftrightarrow{AB}$
$]a , b[$	opened interval	$m(\angle A)$	the measure of the angle A
$]a , b]$	half-opened (closed) interval	$\sim$	similar to
$[a , b[$	half-opened (closed) interval	$>$	greater than or more than
$[a , \infty[$	infinite interval	$\geq$	more than or equal to
$n(a)$	the number of elements of the event a	$<$	smaller than or less than
S	sample space	$\leq$	less than or equal to
$P(a)$	probability of occurring event a		

Sheet (2)
Cartesian Product

The ordered pair

(a , b) is called an ordered pair

- a is called the first projection
- b is called the second projection

Notice that :

1. $(a , b) \neq \{a , b\}$, $(a , b) \neq [a , b]$
2. The element in the ordered pair can be repeated while that cannot happen in the sets.

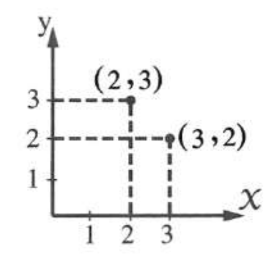
For example :

We say the ordered pair (2 , 2) while we cannot say $\{2 , 2\}$ but we say $\{2\}$

3. There is an empty set which is denoted by ∅ while there is not an empty ordered pair.
4. $(a , b) \neq (b , a)$ where $a \neq b$

For example : $(2 , 3) \neq (3 , 2)$

Notice that : (2 , 3) and (3 , 2) are represented by two different points as shown in the opposite graph.



The equality of two ordered pairs

If $(a , b) = (x , y)$, then $a = x$, $b = y$

For example :

- If $(a , b) = (3 , -4)$, then $a = 3$, $b = -4$
- If $(x , 2) = (-5 , y)$, then $x = -5$, $y = 2$

Example 1

Find the values of x and y in each of the following if :

1. $(x^2 - 1 , 8) = (48 , \sqrt[3]{y})$
2. $(32 , x + y) = (y^5 , 2)$

Solution

1 $\because (x^2 - 1, 8) = (48, \sqrt[3]{y})$

$\therefore x^2 - 1 = 48$ $\quad \therefore x^2 = 49$ $\quad \therefore x = \pm 7$,

$\sqrt[3]{y} = 8$ $\quad \therefore y = 8^3$ $\quad \therefore y = 512$

2 $\because (32, x + y) = (y^5, 2)$ $\quad \therefore y^5 = 32$ $\quad \therefore y = 2$ «for $2^5 = 32$»

, $x + y = 2$ and by substituting by the value of y :

$\therefore x + 2 = 2$ $\quad \therefore x = 0$

The Cartesian product of two finite sets and representing it

For any two finite and non empty sets X and Y, we get :

1 The Cartesian product of the set X by the set Y and it is denoted by $Y \times X$ is the set of all ordered pairs whose first projection of each of them belongs to X and the second projection of each of them belongs to Y **i.e.** $X \times Y = \{(a, b) : a \in X, b \in Y\}$

For example :

If $X = \{1, 2\}$, $Y = \{5, 7, 8\}$, then

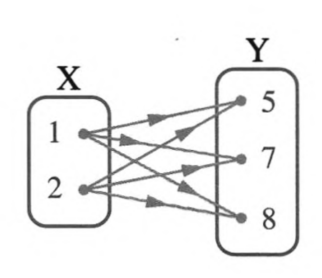
$X \times Y = \{1, 2\} \times \{5, 7, 8\}$

$= \{(1, 5), (1, 7), (1, 8), (2, 5), (2, 7), (2, 8)\}$

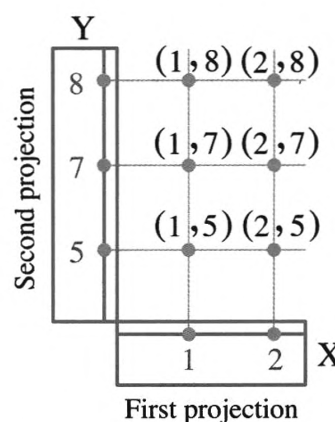
	×	Second projection		
		5	7	8
First projection	1	(1 , 5)	(1 , 7)	(1 , 8)
	2	(2 , 5)	(2 , 7)	(2 , 8)

The opposite table helps us to get $X \times Y$

We can represent $X \times Y$ by an arrow diagram or graphical (Cartesian) diagram as follows :



The arrow diagram



The graphical diagram (The Cartesian diagram)

2 The Cartesian product of the set Y by the set X and which is denoted by $\boxed{Y \times X}$ is the set of all ordered pairs whose first projection belongs to the set Y and the second projection belongs to the set X

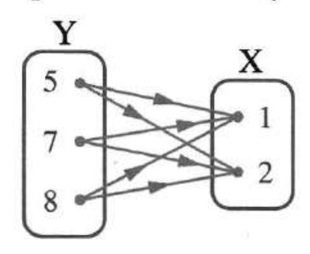
i.e. $Y \times X = \{(a , b) : a \in Y , b \in X\}$

For example :

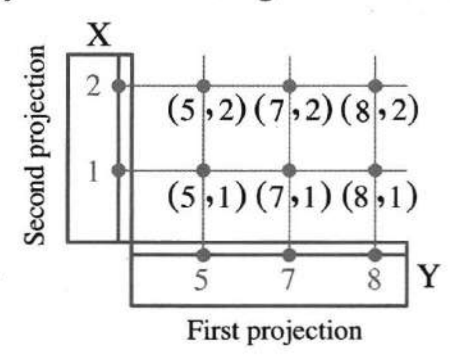
If $X = \{1 , 2\}$, $Y = \{5 , 7 , 8\}$, then $Y \times X = \{5 , 7 , 8\} \times \{1 , 2\}$

$= \{(5 , 1) , (5 , 2) , (7 , 1) , (7 , 2) , (8 , 1) , (8 , 2)\}$

We can represent $Y \times X$ by an arrow diagram or by a Cartesian diagram as follows :



The arrow diagram



The Cartesian diagram

Remarks

From the previous, we notice that :

- **$X \times Y \neq Y \times X$ where $X \neq Y$**

  because $(1 , 5) \neq (5 , 1)$

We say $X \times Y = Y \times X$ at the following cases :

(1) $X = Y$

(2) One of the two sets $= \varnothing$

« $X \times \varnothing = \varnothing \times X = \varnothing$ because $\varnothing$ has no elements »

3 The Cartesian product of the set X by itself and we denote it by $\boxed{X \times X}$ in the same times it is denoted by $\boxed{X^2}$ (it is read X two) is the set of all ordered pairs whose first projections and second projections belong to X

i.e. $X \times X = \{(a , b) : a \in X , b \in X\}$

For example :

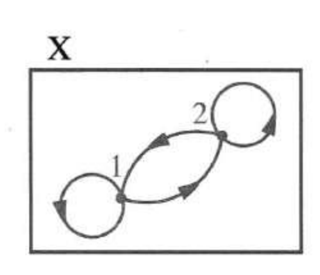
If $X = \{1, 2\}$, then

$X \times X = \{1, 2\} \times \{1, 2\}$

$= \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

	⊗	Second projection	
		1	2
First projection	1	(1 , 1)	(1 , 2)
	2	(2 , 1)	(2 , 2)

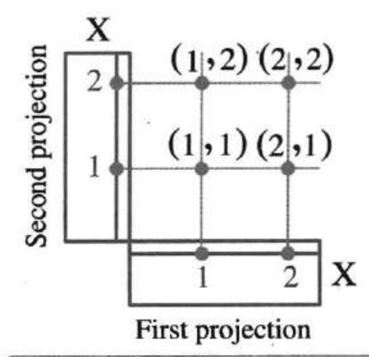
We can represent $X \times X$ by an arrow diagram or Cartesian diagram as follows :



or

X X
1 1
2 2

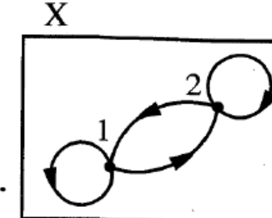
The arrow diagram



Cartesian diagram

Remark

The ordered pairs in which the first projection equals the second projection in the previous Cartesian product (1 , 1) , (2 , 2) are represented in the arrow diagram by a loop ◯ to show that the arrow goes and returns to the same point.



Example 2

If $X = \{2, 3, 4\}$ and $Y = \{a, b\}$ find each of :

1. $X \times Y$
2. $Y \times X$
3. $X \times X$

, then find the number of elements of each of them.

Solution

1. $X \times Y = \{(2, a), (2, b), (3, a), (3, b), (4, a), (4, b)\}$, the number of elements of $X \times Y = 6$ ordered pairs.

2. $Y \times X = \{(a, 2), (a, 3), (a, 4), (b, 2), (b, 3), (b, 4)\}$, the number of elements of $Y \times X = 6$ ordered pairs.

3. $X \times X = \{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4), (4, 2), (4, 3), (4, 4)\}$, the number of elements of $X \times X = 9$ ordered pairs.

Remarks

If we denote the number of elements of any set by «n» then from the previous example, we find that :

- $n(X) = 3$, $n(Y) = 2$

i.e. **1** $n(X \times Y) = n(Y \times X) = n(X) \times n(Y)$

2 $n(X \times X) = (n(X))^2$

Remark

If : $(a, b) \in X \times Y$, then $a \in X$, $b \in Y$

For example :

If : $(3, 5) \in X \times Y$, then $3 \in X$, $5 \in Y$

Example 3

If : $X = \{1, 2\}$, $Y = \{3, 7\}$ and $Z = \{3\}$ **Find :**

(1) $X \times Z$ (2) $n(Y^2)$ (3) $(Y \cap Z) \times X$

Solution

(1) $X \times Z = \{(1, 3), (2, 3)\}$

(2) $n(Y^2) = 2 \times 2 = 4$

(3) $(Y \cap Z) \times X = \{(3, 1), (3, 2)\}$

Example 4

If : $X = \{3, 4\}$, $Y = \{4, 5\}$, $Z = \{6, 5\}$, **then find :**

(1) $X \times (Y \cap Z)$ (2) $(X - Y) \times Z$

Solution

(1) $X \times (Y \cap Z) = \{(3, 5), (4, 5)\}$

(2) $(X - Y) \times Z = \{(3, 6), (3, 5)\}$

Example 5

If : $X = \{3, 4\}$, $Y = \{4, 5\}$, $Z = \{6, 5\}$, **then find :**

(1) $X \times (Y \cap Z)$ (2) $(X - Y) \times Z$

(3) $(X - Y) \times (Y - Z)$

Solution

(1) $X \times (Y \cap Z) = \{(3, 5), (4, 5)\}$

(2) $(X - Y) \times Z = \{(3, 6), (3, 5)\}$

(3) $(X - Y) \times (Y - Z) = \{(3, 4)\}$

The Cartesian product of two infinite sets

As examples for that

$\mathbb{N} \times \mathbb{N} = \{(x , y) : x \in \mathbb{N} , y \in \mathbb{N}\}$, $\mathbb{Z} \times \mathbb{Z} = \{(x , y) : x \in \mathbb{Z} , y \in \mathbb{Z}\}$,

$\mathbb{Q} \times \mathbb{Q} = \{(x , y) : x \in \mathbb{Q} , y \in \mathbb{Q}\}$, $\mathbb{R} \times \mathbb{R} = \{(x , y) : x \in \mathbb{R} , y \in \mathbb{R}\}$

The Cartesian product of two infinite sets

- We know that if X is a finite set (having n elements) , then the Cartesian product X × X is also a finite set (having n^2 elements).

For example: If n (X) = 3 , then n (X × X) = 9

- But if X is an infinite set , then X × X is an infinite set also

As examples for that

$\mathbb{N} \times \mathbb{N} = \{(x , y) : x \in \mathbb{N} , y \in \mathbb{N}\}$, $\mathbb{Z} \times \mathbb{Z} = \{(x , y) : x \in \mathbb{Z} , y \in \mathbb{Z}\}$,

$\mathbb{Q} \times \mathbb{Q} = \{(x , y) : x \in \mathbb{Q} , y \in \mathbb{Q}\}$, $\mathbb{R} \times \mathbb{R} = \{(x , y) : x \in \mathbb{R} , y \in \mathbb{R}\}$

Representing the Cartesian product of two infinite sets

- We know that if X is a finite set , we represent the Cartesian product X × X graphically by a finite number of points.
- But if X is an infinite set , then the Cartesian product X × X represented graphically by an infinite number of points.

First Representing the Cartesian product $\mathbb{N} \times \mathbb{N}$ $(\mathbb{N}^2)$

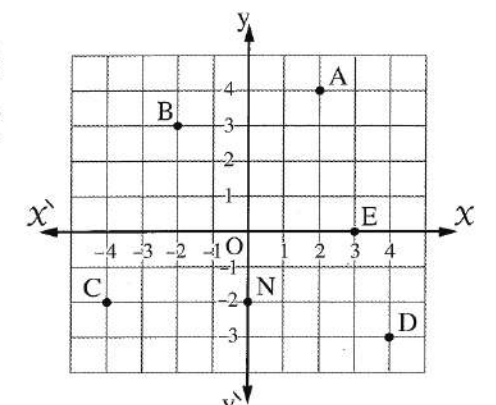
- Represent the natural numbers on two perpendicular straight lines, one of them $\overleftrightarrow{xx}$ is horizontal and the other $\overleftrightarrow{yy}$ is vertical , where they intersect at the point which represents the number zero on each of them *i.e.* O = (0 , 0)
- And the opposite figure shows a small part of the perpendicular graphical net of the Cartesian product $\mathbb{N} \times \mathbb{N}$ which consists of the vertical and the horizontal straight lines that pass through the points which represent the natural numbers on each of $\overleftrightarrow{xx}$ and $\overleftrightarrow{yy}$
- And each point of the points of this net represents an ordered pair of the Cartesian product $\mathbb{N} \times \mathbb{N}$

For example:

- The point A represents the ordered pair (3 , 2)
- The point B represents the ordered pair (5 , 0)
- The point C represents the ordered pair (0 , 4)
- The point O represents the ordered pair (0 , 0)

Second Representing the Cartesian product $\mathbb{Z} \times \mathbb{Z}$ $(\mathbb{Z}^2)$

- Represent the integers on each of $\overleftrightarrow{XX}$ and $\overleftrightarrow{yy}$ which are intersecting at O (0 , 0)
- And the opposite figure shows a small part of the perpendicular graphical net of the Cartesian product $\mathbb{Z} \times \mathbb{Z}$
- And each point of its points represents an ordered pair of the Cartesian product $\mathbb{Z} \times \mathbb{Z}$

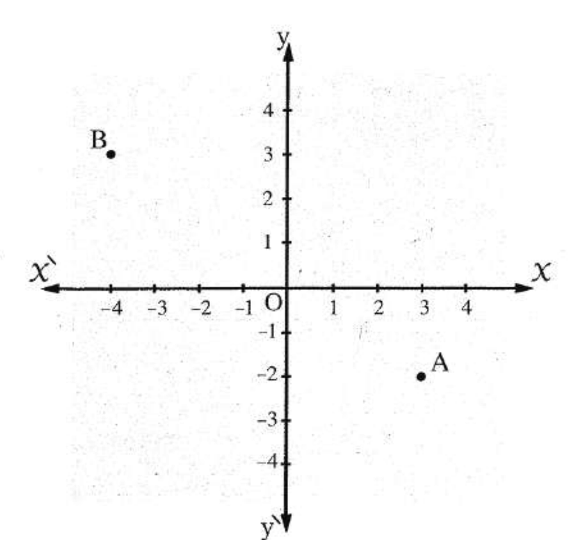


For example:

- The point A represents the ordered pair (2 , 4)
- The point B represents the ordered pair (– 2 , 3)
- The point C represents the ordered pair (– 4 , – 2)
- The point D represents the ordered pair (4 , – 3)
- The point E represents the ordered pair (3 , 0)
- The point N represents the ordered pair (0 , – 2)

Third Representing the Cartesian product $\mathbb{R} \times \mathbb{R}$ $(\mathbb{R}^2)$

- The perpendicular graphical net of the Cartesian product $\mathbb{R} \times \mathbb{R}$ is an infinite extended surface from all sides and the opposite figure shows a small part of this region.
- Each point of this region represents an ordered pair of the Cartesian product $\mathbb{R} \times \mathbb{R}$



For example:

- The point A represents the ordered pair (3 , – 2)
- The point B represents the ordered pair (– 4 , 3)

Remarks

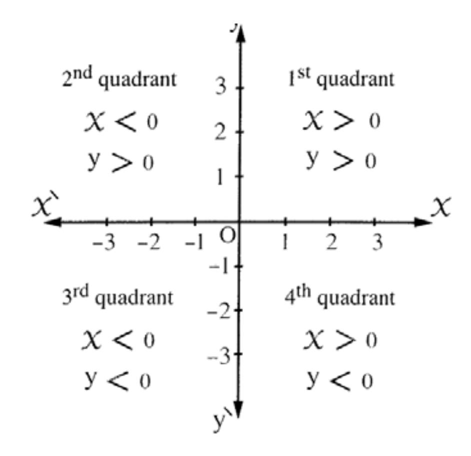
1 The horizontal straight line $\overleftrightarrow{XX}$ is called X-axis or the horizontal axis and the vertical straight line $\overleftrightarrow{yy}$ is called y-axis or the vertical axis.

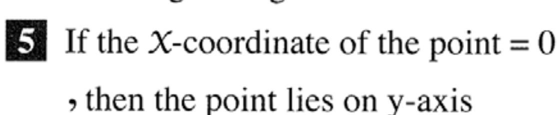
2 The point of intersection of the two axes $\overleftrightarrow{XX}$ and $\overleftrightarrow{yy}$ is called the origin point.

3 If the point A represents the ordered pair (X , y) in the Cartesian product $\mathbb{R} \times \mathbb{R}$, then :

- The first projection X is called the X-coordinate of the point A
- The second projection y is called the y-coordinate of the point A

4 The two axes $\overleftrightarrow{XX'}$ and $\overleftrightarrow{yy'}$ divide the plane into four quadrants as shown in the opposite figure and we can determine the quadrant in which any point lies by knowing the signs of its two coordinates.





5 If the X-coordinate of the point = 0

, then the point lies on y-axis

6 If the y-coordinate of the point = 0

, then the point lies on X-axis

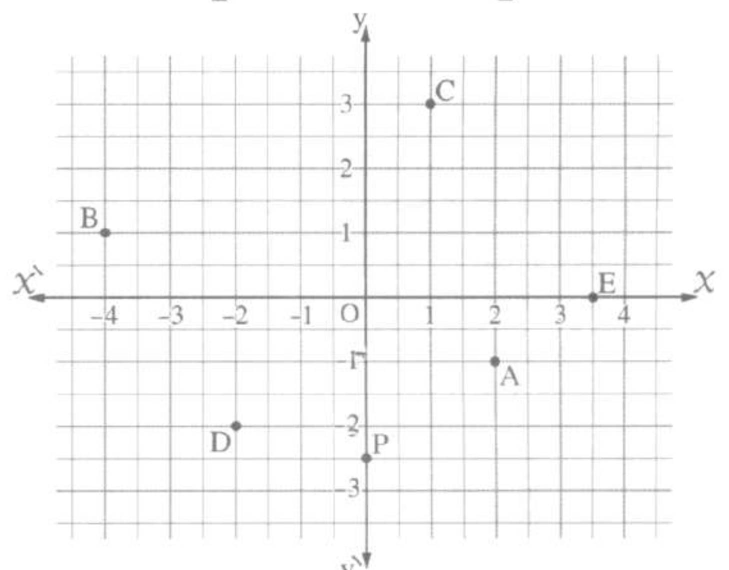
Example 1

Mention the quadrant or the axis on which each of the following points lies on the perpendicular square net of the Cartesian product $\mathbb{R} \times \mathbb{R}$, then locate it on the net :

A (2 , – 1) , B (– 4 , 1) , C (1 , 3) , D (– 2 , – 2) , E $\left(3\frac{1}{2}, 0\right)$, P $\left(0, -2\frac{1}{2}\right)$

Solution

- A (2 , – 1) lies on the 4th quadrant
- B (– 4 , 1) lies on the 2nd quadrant
- C (1 , 3) lies on the 1st quadrant
- D (– 2 , – 2) lies on the 3rd quadrant
- E $\left(3\frac{1}{2}, 0\right)$ lies on the X-axis
- P $\left(0, -2\frac{1}{2}\right)$ lies on the y-axis



Choose the correct answer :

1 If : $(X^3, y + 3) = \left(1, \sqrt{4}\right)$, then : $X - y =$ ....

(a) 3 (b) 2 (c) 1 (d) zero

2 If : (a + 1 , 5) = (– 2 , b – 1) , then : 2 a + b = ... *(El-Ismailia 2014)*

(a) – 12 (b) zero (c) 2 (d) 12

3 If : X = {3} , then : X^2 = ... *(Cairo 2013)*

(a) 9 (b) (3 , 3) (c) {9} (d) {(3 , 3)}

4 If : $X = \{5\}$, $Y = \varnothing$, then $n(X \times Y) =$ ....
(a) 1 (b) 2 (c) 5 (d) zero

5 If : $n(X) = 3$, $Y = \{4, 5\}$, then $n(X \times Y) =$ ....
(a) 2 (b) 3 (c) 5 (d) 6

6 If : $X = \{5, 6, 7\}$, then $n(X^2) =$ ....
(a) 3 (b) 6 (c) 9 (d) 12

7 If $n(X) = 3$, $n(X \times Y) = 12$, then $n(Y) =$ .... *(El-Kalyoubia 2011)*
(a) 4 (b) 9 (c) 15 (d) 36

8 If : $n(X) = 5$, $n(X \times Y) = 15$, then $n(Y) =$ ....
(a) 1 (b) 5 (c) 3 (d) 15

9 If : $X \times Y = \{(1, 3), (1, 4)\}$, then $n(X) =$ ....
(a) 1 (b) 2 (c) 3 (d) 4

10 If : $(3, 5) \in \{2, 6\} \times \{x, 8\}$ then $x =$ ....
(a) 8 (b) 6 (c) 5 (d) 3

11 If : $(X - Y) \times Y = \{(1, 2), (1, 3)\}$ and $n(X \times Y) = 6$, then $X =$ ....
(a) $\{1\}$ (b) $\{1, 2\}$ (c) $\{1, 3, 6\}$ (d) $\{1, 3, 2\}$

12 The point $(-5, 7)$ lies in .... quadrant.
(a) first (b) second (c) third (d) fourth

13 The point $(-3, 4)$ lies in .... quadrant.
(a) first (b) second (c) third (d) fourth

14 The cartesian product $\{2\} \times \mathbb{R}$ represent graphically by a straight line passing through the two points $(2, 0)$ and ....
(a) $(0, 2)$ (b) $(2, 5)$ (c) $(5, 2)$ (d) $(-2, 2)$

15 The point $A(5, -3)$ lies on the .... quadrant.
(a) first (b) second (c) third (d) fourth

16 If the point $(5 , b - 5)$ is located on the X-axis then $b =$ ……………

(a) zero (b) -5 (c) 5 (d) 10

17 If the point $(5 , b - 7)$ is located on the X-axis , then $b =$ ……… *(Alex. 2011)*

(a) 2 (b) 5 (c) 7 (d) 12

18 If : $(|X| , 4) = (3 , y^2)$ and the point (X , y) lies in the second quadrant , then $X + y =$ ……… *(El-Sharkia 2014)*

(a) 7 (b) 1 (c) -1 (d) -7

19 If the point $(X - 5 , 3 - X)$ where $X \in \mathbb{Z}$ is located in the third quadrant , then X equals ……………

(a) 2 (b) 3 (c) 4 (d) 5

Essay problems:

1 **Find :** a , b if $(a - 7 , 26) = (-2 , b^3 - 1)$

2 If $(X - 1 , 11) = (8 , y + 3)$, **then find :** $\sqrt{X + 2y}$

3 If $(X^2 , 27) = (1 , y^3)$ and the point (X , y) lies in the second quadrant , **find the value of :** $\sqrt{y - X}$

4 If $(2X , 4) = (8 , y + 1)$, **then find the value of :** $\sqrt{X^2 + y^2}$

5 If $X = \{2 , 5\}$, $Y = \{1 , 3 , 7\}$, **then find :** (1) $X \times Y$ (2) $n(Y^2)$

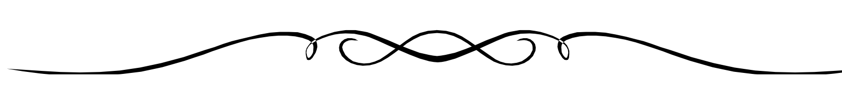
6 If $X = \{3 , 7\}$, $Y = \{1 , 2 , 5\}$ **Find :** $X \times Y$, $n(Y^2)$

7 If $X = \{2 , 3\}$, **then find :** X^2

8	If $X \times Y = \{(1, 1), (1, 3), (1, 5)\}$ **Find :** (1) X , Y (2) $Y \times X$
9	If $X = \{1, 3, 5\}$, $Y = \{4, 5\}$ **Find :** $(X \cap Y) \times (X \cup Y)$
10	If $X \times Y = \{(2, 3), (2, 2), (2, 4)\}$ **Find each of the following :** (1) X , Y (2) $X \times (X \cap Y)$



Homework



Choose the correct answer :

1	If : $(5, x - 8) = (y + 1, -5)$, then : $x + y =$ .... *(Aswan 2011)* (a) 4 (b) 5 (c) 6 (d) 7
2	If : $(2, x - 1) = (y, 0)$, then $x + y =$ .... (a) 3 (b) 1 (c) 2 (d) – 3
3	If : $X = \{3\}$, then : $X^2 =$ .... *(Cairo 2013)* (a) 9 (b) (3 , 3) (c) {9} (d) {(3 , 3)}
4	If : $X = \{5\}$, $Y = \varnothing$, then $n(X \times Y) =$ .... (a) 1 (b) 2 (c) 5 (d) zero
5	If : $n(X) = 2$, $Y = \{1, 2\}$, then $n(X \times Y) =$ .... (a) 4 (b) 3 (c) 5 (d) 6
6	If : $n(X) = 3$, $Y = \{2, 5\}$ then $n(X \times Y) =$ .... (a) 2 (b) 3 (c) 5 (d) 6
7	If : $X = \{3\}$ and $n(Y) = 4$, then $n(X \times Y) =$ .... (a) 1 (b) 4 (c) 7 (d) 12

8 If : $X = \{5\}$, $Y = \{3\}$, then $n(X \times Y) =$ ....

(a) 15 (b) 5 (c) 3 (d) 1

9 If : $X = \{3, 5, 6\}$, then $n(X^2) =$ ....

(a) 3 (b) 6 (c) 9 (d) 12

10 If : $X \times Y = \{(6, 3), (6, 4)\}$, then $n(X) =$ ....

(a) 3 (b) 1 (c) 4 (d) 2

11 📖 If $(3, 5) \in \{3, 6\} \times \{x, 8\}$, then $x =$ .... *(El-Behaira 2011)*

(a) 8 (b) 6 (c) 5 (d) 3

12 The point $(-5, 3)$ lies in .... quadrant.

(a) first (b) second (c) third (d) fourth

13 The point $(-4, 3)$ lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

14 The point $(-2, -5)$ lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

15 If : $(|x|, 4) = (3, y^2)$ and the point (x, y) lies in the second quadrant , then : $x + y =$ ....

(a) 7 (b) 1 (c) -1 (d) -7

16 The cartesian product $\{2\} \times \mathbb{R}$ represent graphically by a straight line passing through the two points $(2, 0)$ and ....

(a) $(0, 2)$ (b) $(2, 5)$ (c) $(5, 2)$ (d) $(-2, 2)$

17 The point $(5, -2)$ lies on the .... quadrant.

(a) first (b) second (c) third (d) fourth

18 If the point $(5 - x, x - 4)$ lies in the fourth quadrant , then the value of $x =$ ....

(a) 9 (b) 8 (c) 6 (d) 2

19	If the point $(x , 2)$ lies on y-axis, then $x =$ .... *(El-Fayoum 2011)* (a) zero (b) 1 (c) 2 (d) 3
20	If the point $(x , 7)$ lies on y-axis, then $x =$ .... *(New vally 2013)* (a) zero (b) 7 (c) -7 (d) 49
21	If : $X = \{5\}$, $Y = \varnothing$, then $n(X \times Y) =$ .... (a) 1 (b) 2 (c) 5 (d) zero

Essay problems:

1	**Find :** a , b if $(a - 7 , 26) = (-2 , b^3 - 1)$
2	If $(x - 1 , 11) = (8 , y + 3)$, **then find :** $\sqrt{x + 2y}$
3	If $(x^2 , 27) = (1 , y^3)$ and the point (x , y) lies in the second quadrant **, find the value of :** $\sqrt{y - x}$
4	If $X = \{2 , 5\}$, $Y = \{1 , 3 , 7\}$ **, then find :** (1) $X \times Y$ (2) $n(Y^2)$
5	If $X \times Y = \{(1 , 2) , (1 , 3) , (2 , 2) , (2 , 3)\}$ **Then find :** (1) $X \cup Y$ (2) Y^2
6	If $X = \{2 , 15\}$, $Y = \{4 , 1\}$ and $Z = \{15\}$ **Find :** (1) $Y \times Z$ (2) $n(X^2)$ (3) $(X \cap Z) \times Y$
7	If $X = \{1\}$, $Y = \{2 , 3\}$, $Z = \{2 , 5 , 6\}$ **Find :** the Cartesian product $(Z - Y) \times (X \cup Y)$
8	If : $X = \{1 , 2 , 6\}$, $Y = \{2 , 4 , 5 , 6\}$ and $Z = \{4\}$ **Find :** (1) $X \times Y$ (2) $(X \cap Y) \times Z$
9	If $X = \{1 , 2\}$, $Y = \{2 , 5\}$, $Z = \{4 , 5\}$, then : **Find :** $(X \cap Y) \times (Y \cup Z)$

Sheet (3)

Relation - Function (mapping)

Sheet (3) Relation - Function (mapping)

First The relation

Remarks

1 The relation R is a subset of the Cartesian product $X \times Y$ *i.e.* $R \subset X \times Y$

2 If $(a , b) \in$ the relation R , then we can express that by another method , we write "a R b" , it means that the element a is connected with the element b by the relation R

The conclusion

1 The relation from a set X to a set Y is a connection joining some or all the elements of X with some or all the elements of Y

2 If R is a relation from the set X to the set Y , then R is a set of ordered pairs where the first projection of each belongs to X and the second projection belongs to Y and the first projection connects with the second projection with respect to this relation.

3 The relation R from the set X to the set Y is a subset from the Cartesian product $X \times Y$

i.e. The relation $R \subset X \times Y$

Inversely : any subset of the Cartesian product $X \times Y$ expresses a relation from X to Y

4 The relation can be represented by an arrow diagram or by a Cartesian diagram (graphically).

Remark

If R is a relation from X to X , then : R is a relation on X and the relation $R \subset X \times X$

Second Functions (Mapping)

Generally

A relation from X to Y is said to be a function if :

1 In the relation , each element of the set X appears only once as a first projection in one of the ordered pairs of the relation. (Notice the relation R in the previous example)

2 In the arrow diagram which represents the relation , each element of X has one and only one arrow going out of it to one element of Y

(Notice the arrow diagram of the previous relation)

3 In the Cartesian diagram which represents the relation , each vertical line has one and only one point lying on it of the points which represent the relation.

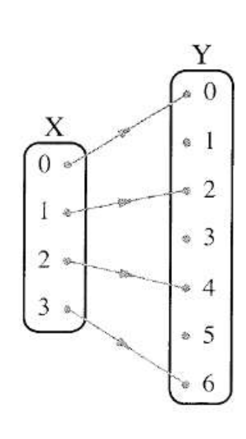
(Notice the Cartesian diagram of the previous relation)

Introductory example

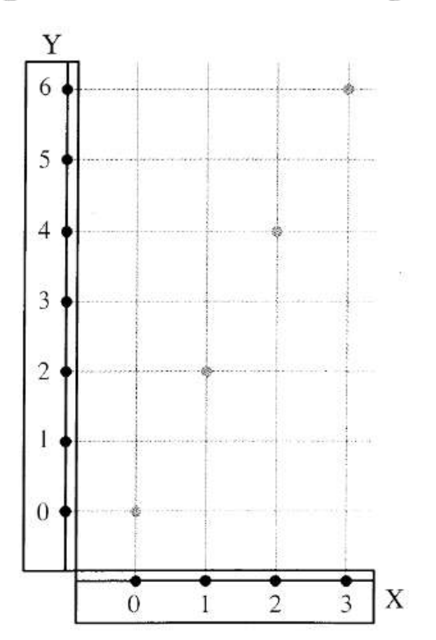
If $X = \{0, 1, 2, 3\}$, $Y = \{0, 1, 2, 3, 4, 5, 6\}$ and R is a relation from X to Y where "a R b" means "$a = \frac{1}{2} b$" for each $a \in X$, $b \in Y$

Write R and represent it by an arrow diagram and a Cartesian diagram.

Solution $R = \{(0, 0), (1, 2), (2, 4), (3, 6)\}$



The arrow diagram



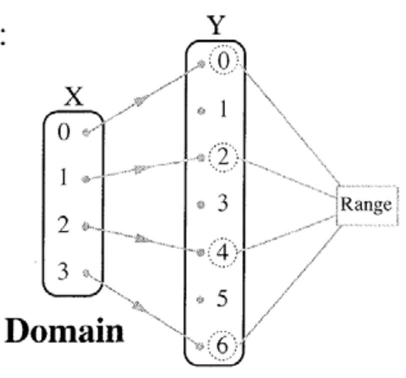
The Cartesian diagram

In the previous relation, we notice that :

Each element of the set X has been connected with **one and only one element** of the elements of the set Y

Such as, this relation is called a function or (mapping), also :

- The set of $X = \{0, 1, 2, 3\}$ is called "the domain of the function".
- The set of $Y = \{0, 1, 2, 3, 4, 5, 6\}$ is called "the codomain of the function".
- The set $\{0, 2, 4, 6\}$ is called "the range of the function" and it is a subset from the codomain of the function.



Prime numbers = { 2 , 3 , 5 , 7 , 11 , 13 , 17 , 19 , 23 , 29 , 31 , 37 }

Odd numbers = { 1 , 3 , 5 , 7 , 9 , 11 , 13 , 15 , 17 , 19 , ... }

Even numbers = { 0 , 2 , 4 , 6 , 8 , 10 , 12 , 14 , 16 , 18 , 20 , ... }

Example 8 If $X = \{0, 1, 2, 3\}$, $Y = \{2, 3, 4, 5, 6\}$ and R is a relation from X to Y where "a R b" means "a + b = 5" for each $a \in X$, $b \in Y$

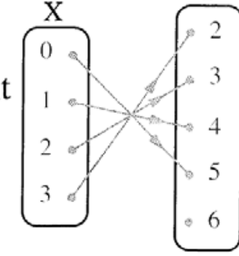
Write the relation R and represent it by an arrow diagram.

Mention giving reasons if R is a function from X to Y or not ?

And if it is a function, find its range.

Solution

- $R = \{(0, 5), (1, 4), (2, 3), (3, 2)\}$



R represents a function from X to Y because each element of X connects with only one element of Y

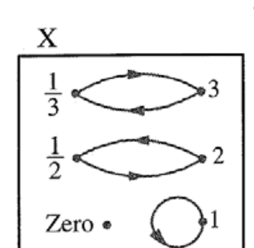
The range of the function $= \{5, 4, 3, 2\}$

Example 9 If $X = \{3, 2, 1, 0, \frac{1}{2}, \frac{1}{3}\}$, and R is a relation on X where "a R b" means "a is the multiplicative inverse of b" for each $a \in X$, $b \in X$

Write R and represent it by an arrow diagram and mention giving reasons if R represents a function or not.

Solution

- $R = \{(3, \frac{1}{3}), (2, \frac{1}{2}), (1, 1), (\frac{1}{2}, 2), (\frac{1}{3}, 3)\}$



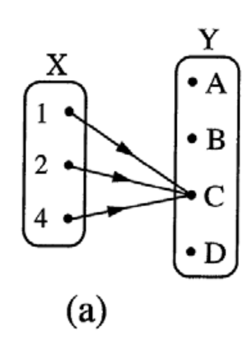
R does not represent a function because the element zero $\in X$ does not connect with any element in X (There is no arrow going out from zero in the arrow diagram which represents the relation)

Choose the correct answer :

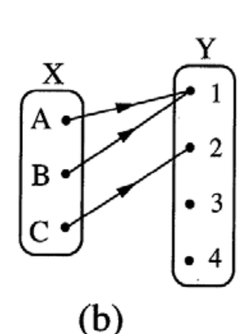
1 If f is a function from the set X to the set Y then the domain of f is ....

(a) X (b) Y (c) $X \times Y$ (d) $Y \times X$

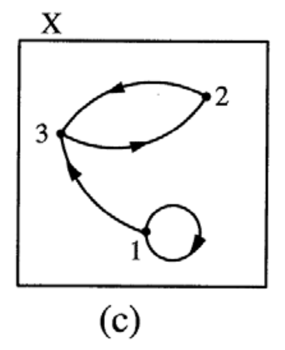
2 The following figures shows four arrow diagrams one of them is not function it is .... *(El-Dakahlia 2014)*



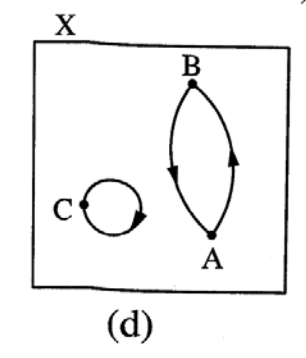
(a)



(b)



(c)



(d)

3 If : $(2 , x - 1) = (y , 0)$, then $x + y =$ ....

(a) 3 (b) 1 (c) 2 (d) -3

4 If : $(x^3 , y + 3) = (1 , \sqrt{4})$, then : $x - y =$ ....

(a) 3 (b) 2 (c) 1 (d) zero

5 If : $n(X) = 2$, $Y = \{1 , 2\}$, then $n(X \times Y) =$ ....

(a) 4 (b) 3 (c) 5 (d) 6

6 If : $X = \{5\}$, $Y = \{3\}$, then $n(X \times Y) =$ ....

(a) 15 (b) 5 (c) 3 (d) 1

7 If : $n(X) = 5$, $n(X \times Y) = 15$, then $n(Y) =$ ....

(a) 1 (b) 5 (c) 3 (d) 15

8 If : $X \times Y = \{(1 , 3) , (1 , 4)\}$, then $n(X) =$ ....

(a) 1 (b) 2 (c) 3 (d) 4

9 If : $(X - Y) \times Y = \{(1 , 2) , (1 , 3)\}$ and $n(X \times Y) = 6$, then $X =$ ....

(a) $\{1\}$ (b) $\{1 , 2\}$ (c) $\{1 , 3 , 6\}$ (d) $\{1 , 3 , 2\}$

10 The point $(-3 , 4)$ lies in .... quadrant.

(a) first (b) second (c) third (d) fourth

11 If the point $(3 , b)$ lies on the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$, where $f(X) = 5X + 4$, then $b =$ ....

(a) 5 (b) 4 (c) 0 (d) 19

12 The point $A (5 , -3)$ lies on the .... quadrant.

(a) first (b) second (c) third (d) fourth

13 If the point $(5 - X , X - 4)$ lies in the fourth quadrant , then the value of $X =$ ....

(a) 9 (b) 8 (c) 6 (d) 2

14 If the point $(X - 5 , 3 - X)$ where $X \in \mathbb{Z}$ is located in the third quadrant , then X equals ....

(a) 2 (b) 3 (c) 4 (d) 5

15 If : $X = \{1 , 3 , 5\}$ and R is function on X where $R = \{(a , 3) , (b , 1) , (1 , 5)\}$, then the numerical value of the expression : $a + b =$ ....

(a) 3 (b) 4 (c) 5 (d) 8

16 The opposite figure represents a function on X , its range = ...

X
b
c
a

(a) $\{a\}$ (b) $\{a , b , c\}$

(c) $\{a , b\}$ (d) $\{b , c\}$

17 If : $(a + 1 , 5) = (-2 , b - 1)$, then : $2a + b =$ .... *(El-Ismailia 2014)*

(a) -12 (b) zero (c) 2 (d) 12

18 If : $n(X) = 3$, $Y = \{2 , 5\}$ then $n(X \times Y) =$ ....

(a) 2 (b) 3 (c) 5 (d) 6

19 If : $X = \{5 , 6 , 7\}$, then $n(X^2) =$ ....

(a) 3 (b) 6 (c) 9 (d) 12

20 If : $X \times Y = \{(6 , 3) , (6 , 4)\}$, then n (X) = ...

(a) 3 (b) 1 (c) 4 (d) 2

21 The point (– 5 , 7) lies in .... quadrant.

(a) first (b) second (c) third (d) fourth

22 The point (– 2 , – 5) lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

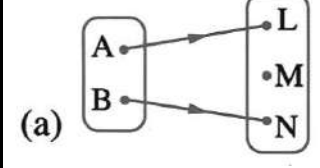
23 The point (5 , – 2) lies on the .... quadrant.

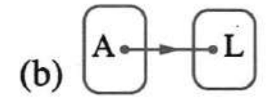
(a) first (b) second (c) third (d) fourth

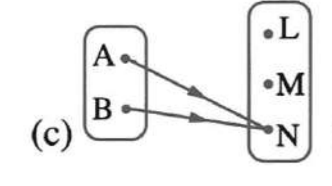
24 If the point (X , 2) lies on y-axis, then X = ... *(El-Fayoum 2011)*

(a) zero (b) 1 (c) 2 (d) 3

25 Which of the following relations does not represent a function from X to Y ? *(Helwan 2011)*

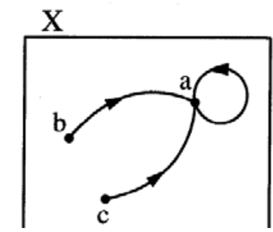
(a) 

(b) 

(c) 

(d) A • → • L ; B •

26 The opposite diagram represents a function on X , its range is .... *(Cairo 2011)*



(a) {a} (b) {a , b , c}

(c) {a , b} (d) {b , c}

27 If : X = {3} , then : X^2 = ... *(Cairo 2013)*

(a) 9 (b) (3 , 3) (c) {9} (d) {(3 , 3)}

Essay problems:

1	If : $X = \{1, 3, 4\}$, $Y = \{1, 2, 3, 4, 5\}$ and R is a relation from X to Y where "a R b" means "$a + b = 6$" for all $a \in X$, $b \in Y$, Write R and represent it by an arrow diagram. Is R a function ? and why ?
2	If : $X = \{1, 2, 5\}$ and R is a relation on X where aRb means "$a + 2b$ = an odd number" for each $a \in X$ and $b \in X$. Write R and represent it by an arrow diagram. Is R a function ?
3	If : $X = \{2, 3, 6\}$, $Y = \{3, 4, 5, 6, 7, 8\}$, $R : X \longrightarrow Y$ Such that : $R(x) = 9 - x$ (1) Find the set of images of elements of the set X by the function R (2) Is R a linear function ? "State reason".
4	If : $X = \{-1, 0, 1, 2, 3\}$, $Y = \{0, 1, 4, 6, 9\}$ and R is a relation from X to Y where "aRb" means "$a^2 = b$" for each of $a \in X$, $b \in Y$ Write R and represent it by a cartesian diagram.
5	If $X = \{1, 2, 3\}$, $Y = \{1, 3, 6, 9, 12\}$ and R is a relation from X to Y, where "a R b" means "$a = \frac{1}{3} b$" for each $a \in X$, $b \in Y$ Write R and show that R is a function, then write its range. (Assiut 2011)
6	If the function $f = \{(0, 4), (1, 3), (2, 2), (3, 1)\}$ 1 Write each of domain and range of the function f 2 Write the rule of the function f (Cairo 2013)
7	If : $f : X \longrightarrow Y$, $X = \{-1, 2, 3\}$, $Y = \{2, 3, 5, 7\}$ $R = \{(-1, 3), (3, 5), (2, 7)\}$ **Find :** (1) The domain of the function f (2) The codomain of the function f (3) The range of the function f



Homework

Choose the correct answer :

1 If : $n(X) = 3$, $Y = \{4 , 5\}$, then $n(X \times Y) =$ ....
(a) 2 (b) 3 (c) 5 (d) 6

2 If : $X = \{3 , 5 , 6\}$, then $n(X^2) =$ ....
(a) 3 (b) 6 (c) 9 (d) 12

3 If $(3 , 5) \in \{3 , 6\} \times \{x , 8\}$, then $x =$ .... *(El-Behaira 2011)*
(a) 8 (b) 6 (c) 5 (d) 3

4 The point $(-5 , 3)$ lies in .... quadrant.
(a) first (b) second (c) third (d) fourth

5 If : $(|x| , 4) = (3 , y^2)$ and the point (x , y) lies in the second quadrant , then : $x + y =$ ....
(a) 7 (b) 1 (c) -1 (d) -7

6 If the point $(5 , b - 5)$ is located on the x-axis then $b =$ ....
(a) zero (b) -5 (c) 5 (d) 10

7 If the point $(x , 7)$ lies on y-axis, then $x =$ .... *(New vally 2013)*
(a) zero (b) 7 (c) -7 (d) 49

8 The set of images of the elements of the domain of the function is called ....
(a) the rule. (b) the domain. (c) the range. (d) the codomain.

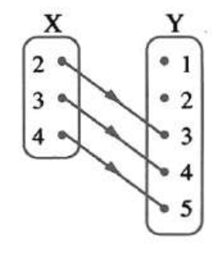
9 If : $(5 , x - 8) = (y + 1 , -5)$, then : $x + y =$ .... *(Aswan 2011)*
(a) 4 (b) 5 (c) 6 (d) 7

10 If : $X = \{5\}$, $Y = \varnothing$, then $n(X \times Y) =$ ....
(a) 1 (b) 2 (c) 5 (d) zero

11 If : $X = \{3\}$ and $n(Y) = 4$, then $n(X \times Y) =$ ....

(a) 1 (b) 4 (c) 7 (d) 12

12 The opposite diagram represents a function from X to Y , then the range = ... (Giza 2011)



(a) $\{2, 3, 4\}$ (b) $\{2, 3, 5\}$

(c) $\{3, 4, 5\}$ (d) Y

13 If $n(X) = 3$, $n(X \times Y) = 12$, then $n(Y) =$ .... *(El-Kalyoubia 2011)*

(a) 4 (b) 9 (c) 15 (d) 36

14 If : $(3, 5) \in \{2, 6\} \times \{x, 8\}$ then $x =$ ....

(a) 8 (b) 6 (c) 5 (d) 3

15 The point $(-4, 3)$ lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

16 The cartesian product $\{2\} \times \mathbb{R}$ represent graphically by a straight line passing through the two points $(2, 0)$ and ....

(a) $(0, 2)$ (b) $(2, 5)$ (c) $(5, 2)$ (d) $(-2, 2)$

17 If the point $(5, b - 7)$ is located on the x-axis , then $b =$ .... *(Alex. 2011)*

(a) 2 (b) 5 (c) 7 (d) 12

18 The point $(-2, -5)$ lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

19 The point $(5, -2)$ lies on the .... quadrant.

(a) first (b) second (c) third (d) fourth

20 If the point $(x, 2)$ lies on y-axis , then $x =$ .... *(El-Fayoum 2011)*

(a) zero (b) 1 (c) 2 (d) 3

Essay problems:

1	If : $X = \{1, 3, 4, 5\}$, $Y = \{1, 2, 3, 4, 5, 6\}$ and R is a relation from X to Y where "a R b" means "$a + b = 7$" for all $a \in X$, $b \in Y$ Write R and represent it by an arrow diagram, show that R is a function ? Write its range.
2	If $X = \{3, 4, 5\}$, $Y = \{1, 5, 4, 6\}$ and R is a relation from X to Y where "a R b" means "$a + b = 9$" for each $a \in X$ and $b \in Y$, write R and represent it by an arrow diagram and show that R is a function , write its range.
3	If : $X = \{2, 3, 6\}$, $Y = \{3, 4, 5, 6, 7, 8\}$, $R : X \longrightarrow Y$ Such that : $R(x) = 9 - x$ (1) Find the set of images of elements of the set X by the function R (2) Is R a linear function ? "State reason".
4	📖 If $X = \{0, 1, 4, 7\}$, $Y = \{1, 3, 5, 6\}$ and R is a relation from X to Y where "a R b" means "$a + b < 8$" for each $a \in X$, $b \in Y$. Write R and represent it by an arrow diagram and also by a Cartesian diagram. Is R a function ? and why ? (El–Kalyoubia 2011)
5	If : $X = \{2, 3, 4, 7\}$, $Y = \{1, 2, 3, 4, 7, 8\}$ and R is a relation from X to Y where "aRb" means "$a - b$ is a prime number" for all $a \in X$, $b \in Y$ write R and represent it by an arrow diagram.
6	If : $X = \left\{\frac{1}{3}, \frac{1}{2}, 1, 2, 3\right\}$ and R is a relation on X where "aRb" means "$ab = 1$" for every $a \in X$, $b \in X$, represent R by an arrow diagram, show that R is a function and write its range.
7	If : $X = \{2, 3, 4\}$, $Y = \{y : y \in \mathbb{N}, 2 \le y < 9\}$ where $\mathbb{N}$ is a set of the natural numbers , and R is a relation from X to Y where "a R b" means : "$a = \frac{1}{2} b$" for all $a \in X$ and $b \in Y$, write R and represent it by an arrow diagram. Show that R is a function from X to Y and write its range.
8	If : $X = \{-1, 0, 1, 2, 3\}$, $Y = \{0, 1, 4, 6, 9\}$ and R is a relation from X to Y where "aRb" means "$a^2 = b$" for each of $a \in X$, $b \in Y$ Write R and represent it by a cartesian diagram.

9

If : $X = \{1, 2, 4\}$ and R is a relation on X where "a R b" means "a is twice of b" for each $a \in X$, $b \in X$

1 Write R and represent it by an arrow diagram.

2 Is R a function ? and why ? *(North Sinai 2013)*

10

If : $X = \{0, 1, 2, 3\}$, $Y = \{-3, -2, -1, 0\}$ and R is a relation from X to Y where "aRb" means that the number "a is the additive inverse to the number b" for every $a \in X$ and $b \in Y$

(1) Find the relation R

(2) Represent the relation R by an arrow diagram.

(3) Is R a function ? Why ?

11

If function $f = \{(1, 3), (2, 5), (3, 7), (4, 9), (5, 11)\}$

(1) Write each of domain and range of f

(2) Write the rule of the function f

12

If : $X = \{1, 3, 5\}$ and R is a function on X where $R = \{(a, 3), (b, 1), (1, 5)\}$ Find the numerical value of a + b

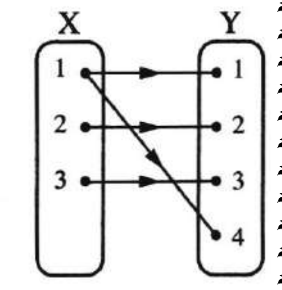
13

If : $X = \{1, 3, 5\}$, R is a function on X, $R = \{(a, 3), (b, 1), (1, 5)\}$, then find the numerical value of the expression : a + b *(Cairo 2014)* « 8 »

14

The opposite arrow diagram represents the relation R from the set X to the set Y, where $X = \{1, 2, 3\}$, $Y = \{1, 2, 3, 4\}$

Write R, is R a function ? why ?



Sheet (4)

Polynomial Functions Part (1)

Sheet (4)
Polynomial Functions Part (1)

The symbolic representation of the function

- The function is usually denoted by one of the following letters. f or m or q or ... and the function f from the set X to the set Y is written mathematically as :

  $f : X \longrightarrow Y$ and is read as **f is a function from X to Y**

  or **m : X** $\longrightarrow$ **Y** and is read as **m is a function from X to Y** and so on ...

- If the ordered pair (x , y) belongs to the function , then the element y is called the image of the element x by the function f and we express it by one of the following two forms :

  $f : x \longmapsto y$ it is read as **f maps x to y**

  or $f : f(x) = y$ it is read as **f is a function where $f(x) = y$**

For example:

If $f : X \longrightarrow Y$ where $f : x \longmapsto x^2$, then $f : 3 \longmapsto 9$

, also can be written in the form : $f(x) = x^2$, hence $f(3) = 9$

Remark

The mathematical form $f(x) = x^2$ is called the rule of the function f, and it is used to find the image of any element of the domain by the function f

Remember that :

- **If f is a function from the set X to the set Y i.e. $f : X \longrightarrow Y$, then :**

1. X is called the **domain** of the function f
2. Y is called the **codomain** of the function f
3. The set of images of the elements of the set X by the function f is called the **range** of the function f which is a subset of the codomain Y

Polynomial functions

Definition

The function $f : R \longrightarrow R$, $f(x) = a_0 + a_1 x + a_2 x^2 + \cdots\cdots + a_n x^n$ where $a_0 , a_1 , a_2 , \cdots\cdots , a_n \in \mathbb{R}$, $n \in \mathbb{N}$ is called a polynomial function.

i.e. The polynomial function is a function whose rule is a term or an algebraic expression in condition that the following should be identified :

1. Each of the domain and the codomain of the function is the set of real numbers.
2. The power (the index) of the variable x in any of its terms is a natural number.

For example: The following functions are all polynomial functions :

- $f : f(X) = 2X + 5$
- $g : g(X) = X^2 - 2X + 1$
- $k : k(X) = 8$
- $n : n(X) = 1 + \sqrt{2}X - 9X^3$

Remark

If the domain or the codomain of a function is not the set of real numbers , then that function is not a polynomial function.

For example:

- $f : f(X) = \sqrt{X}$ is not a polynomial function because $f(X)$ doesn't exist in $\mathbb{R}$ if X equals a negative number.

For example:

$f(-1) \notin \mathbb{R}$ because $\sqrt{-1} \notin \mathbb{R}$, so the domain of the function f is not the set of real numbers.

- $h : h(X) = \frac{1}{X}$ is not a polynomial function

  because $h(X)$ doesn't exist in $\mathbb{R}$ if X equals zero.

i.e. $h(0) \notin \mathbb{R}$

, so the domain of the function h is not the set of real numbers.

Remark

When we search if the function is a polynomial or not , we do not simplify its rule.

For example:

The function $f_1 : f_1(X) = X\left(X + \frac{1}{X}\right)$ doesn't represent a polynomial function because $f_1(0) \notin \mathbb{R}$ while the function $f_2 : f_2(X) = X^2 + 1$ represents a polynomial function.

And notice that: $X\left(X + \frac{1}{X}\right) = X^2 + 1$ for all real numbers except 0

The degree of the polynomial function

The degree of the polynomial function is the highest power of the variable in the function rule.

For example:

- The function $f_1 : f_1(X) = 3X - \frac{1}{2}$ is of the first degree (a linear function)
- The function $f_2 : f_2(X) = \sqrt{5}X^2 - 3X + 4$ is of the second degree (quadratic function)
- The function $f_3 : f_3(X) = X^3 - 5X^2 + 4$ is of the third degree (cubic function)

Remark

The function $f : f(X) = a$ where $a \in \mathbb{R} - \{0\}$
is a polynomial function of zero degree. (a constant function) as $f(X) = 3$
In the case of $a = 0$ *i.e.* when $f(X) = 0$, then the function has no degree.

Example (3) **If $f : \mathbb{R} \longrightarrow \mathbb{R}$, mention the degree of f in each of the following :**

1 $f(x) = 5 - 3x$

2 $f(x) = 3x - x^2$

3 $f(x) = 5x - 3x^2 + x^3$

4 $f(x) = x^2 (2 + x)^2$

Solution

1 f is of the first degree.

2 f is of the second degree.

3 f is of the third degree.

4 $\because f(x) = x^2 (4 + 4x + x^2)$

$= 4x^2 + 4x^3 + x^4$

$\therefore f$ is a function of the fourth degree.

Notice that :

- When we want to determine the degree of the function we should simplify its rule to the simplest form before telling its degree.

Example (4) **If $f : f(x) = x^2 - 2x + 5$**

1 **Find :** $f(1)$, $f(0)$, $f(-2)$, $f\left(\frac{1}{2}\right)$ and $f\left(\sqrt{5}\right)$

Solution

1 $f(1) = (1)^2 - 2 \times (1) + 5 = 1 - 2 + 5 = 4$

Similarly, $f(0) = 5$, $f(-2) = 13$

, $f\left(\frac{1}{2}\right) = 4\frac{1}{4}$ and $f\left(\sqrt{5}\right) = 10 - 2\sqrt{5}$

Example (5) **If $f(x) = 2x + b$ and $g(x) = x^2 + b$ and if $f(2) + g(-4) = 30$, then find : $f(-2) - g(2)$**

Solution

$\because f(2) = 2 \times 2 + b = 4 + b$, $g(-4) = (-4)^2 + b = 16 + b$

, $\because f(2) + g(-4) = 30$		$\therefore 4 + b + 16 + b = 30$

$\therefore 20 + 2b = 30$		$\therefore 2b = 30 - 20 = 10$

$\therefore b = \frac{10}{2} = 5$

$\therefore f(x) = 2x + 5$, $g(x) = x^2 + 5$

$\therefore f(-2) = 2 \times (-2) + 5 = 1$, $g(2) = 2^2 + 5 = 9$

$\therefore f(-2) - g(2) = 1 - 9 = -8$

Choose the correct answer :

1 The function $f : f(X) = (X-5)^3$ is a polynomial function of .... degree. (Qena 2011)

(a) zero (b) second (c) third (d) fourth

2 The function : $f(X) = X(X^2-3)$ is of .... degree.

(a) 1^{th} (b) 2^{th} (c) 3^{th} (d) 4^{th}

3 The function f where $f(X) = X^4 - 2X^3 + 5$ is a polynomial function of .... degree. (Cairo 2014)

(a) first (b) second (c) third (d) fourth

4 The function f where $f(X) = 2X - 3X^4 + 1$ is a polynominal function of .... degree. (El-Sharkia 2011)

(a) first (b) second (c) third (d) fourth

5 The function f where $f(X) = 6X^7 + 2X^5 - 4X + 1$ is a polynomial function of .... degree. (Kafr El-Sheikh 2011)

(a) first (b) fifth (c) sixth (d) seventh

6 If : $f(X) = X + 3$, then $f(-2) =$ ....

(a) 9 (b) -3 (c) 1 (d) 5

7 If : $f(X) = 5X - 7$, then $f(3) =$ ....

(a) 2 (b) 3 (c) 8 (d) 15

8 If : $f(X) = 5X - 3$, then $f(0) =$ ....

(a) 5 (b) 2 (c) 3 (d) -3

9 If : $f(X) = 7X - \frac{1}{2}$, then $f\left(\frac{1}{2}\right) =$ ....

(a) 7 (b) $\frac{1}{2}$ (c) 3 (d) $\frac{7}{2}$

10 If : $f(X) = X^2 - \sqrt{2}X$, then : $f(\sqrt{2}) =$ .... (El-Dakahlia 2011)

(a) 4 (b) 2 (c) 6 (d) zero

11 If : $f(X) = X^2 - X + 3$, then : $f(3) =$ .... (Beni Suef 2011)

(a) 3 (b) 6 (c) 9 (d) 12

Essay problems:

1	If : $f(X) = 2X - 1$, prove that : $f(2) - 3f(1) = \text{zero}$ (El-Gharbia 2011)
2	If : $f(X) = 2X^2 - 5X + 2$, **then prove that :** $f(2) = f\left(\frac{1}{2}\right)$ ***(Luxor 2014)***
3	If : $f(X) = 2X - 1$, **then prove that :** $f(2) - 3f(1) = \text{zero}$ ***(El-Gharbia 2011)***
4	**If : $f(X) = X^2 - X + 3$, then find : $f(-2)$, $f(0)$, $f(1)$**
5	**Which of the following functions represents a polynomial function :** $f : f(X) = X^2 + \sqrt{X} + 8$
6	**Which of the following functions represents a polynomial function :** $f : f(X) = X^3 + X^2 + 3$
7	**Which of the following functions represents a polynomial function :** $f : f(X) = X\left(X + \frac{1}{X} - 2\right)$
8	If $f(X) = 2X^2 - 5X + 2$, **then prove that :** $f(2) = f\left(\frac{1}{2}\right)$ *(Luxor 2014)*
9	If $f(X) = 2X - 1$, **then prove that :** $f(2) - 3f(1) = \text{zero}$ *(El-Gharbia 2011)*
10	**If f is a function on X where X = {3 , 4 , 5 , 6} and $f(3) = 3$, $f(4) = 5$, $f(5) = 5$, $f(6) = 5$** (1) Represent f by an arrow diagram. (2) Write the set of f and mention its range. *(Ismailia 2015)*
11	If the function $f = \{(0, 4), (1, 3), (2, 2), (3, 1)\}$ (1) Write each of domain and range of the function f (2) Write the rule of the function f *(Cairo 2013)*



Homework

Choose the correct answer :

1 If : $f(x) = x^2 + 7$ then $f(3) =$ ....

(a) 10 (b) 7 (c) 9 (d) 16

2 If : $f(x) = a\,x + 6$, $f(2) = 2$, then a = ... *(New Valley 2006)*

(a) 2 (b) – 2 (c) 4 (d) 6

3 If : $f(x) = 4\,x + b$, $f(3) = 15$, then b = ...

(a) 156 (b) 3 (c) 4 (d) – 3

4 If : $(2, -6) \in$ the set of the function f where $f(x) = m\,x + 8$, then m = ...

(a) – 16 (b) 7 (c) – 7 (d) 2

5 If : $(a, a) \in$ the function f where $f(x) = 2\,x - 3$, then a = ...

(a) 3 (b) 2 (c) 1 (d) zero

6 If : $(a, a) \in$ the function f where $f(x) = 4\,x - 6$, then a = ...

(a) 2 (b) 3 (c) 1 (d) zero

7 The set of images of the elements of the domain of the function is called .... *(Damietta 2015)*

(a) the rule (b) the domain (c) the range (d) the codomain

8 The function f where $f(x) = x^4 - 2\,x^3 + 7$
is a polynomial function of .... degree. *(Suez 2015)*

(a) first (b) second (c) third (d) fourth

9 The function $f : f(x) = (x - 5)^3$ is a polynomial function of .... degree. *(Qena 2011)*

(a) zero (b) second (c) third (d) fourth

10 If $f(x) = x^2 - \sqrt{2}\,x$, then $f\left(\sqrt{2}\right) =$ .... *(El-Dakahlia 2011)*

(a) 4 (b) 2 (c) 6 (d) zero

11 If $f(x) = k\,x + 8$, $f(2) = 0$, then k = ... *(El-Sharkia 2015)*

(a) 8 (b) 6 (c) 4 (d) – 4

Essay problems:

1	If : $f(X) = X^2 - X + 3$, **then find :** $f(-2)$, $f(0)$, $f(1)$
2	If the function $f = \{(0, 4), (1, 3), (2, 2), (3, 1)\}$ (1) Write each of domain and range of the function f (2) Write the rule of the function f *(Cairo 2013)*
3	If the set of the function $f = \{(1, 3), (2, 5), (3, 7), (4, 9), (5, 11)\}$ (1) Write the domain and the range of the function f (2) Write the rule of the function f *(Red Sea 2015)*
4	**Which of the following functions represents a polynomial function :** 📖 $f : f(X) = X\left(X + \frac{1}{X} - 2\right)$
5	**If f is a function on X where X = {3 , 4 , 5 , 6} and $f(3) = 3$, $f(4) = 5$, $f(5) = 5$, $f(6) = 5$** (1) Represent f by an arrow diagram. (2) Write the set of f and mention its range. *(Ismailia 2015)*
6	**Which of the following functions represents a polynomial function :** 📖 $f : f(X) = X^3 + X^2 + 3$
7	If $f(X) = 2X - 1$, **then prove that :** $f(2) - 3f(1) = \text{zero}$ *(El-Gharbia 2011)*
8	**Which of the following functions represents a polynomial function :** 📖 $f : f(X) = X^2 + \sqrt{X} + 8$
9	If $f(X) = 2X^2 - 5X + 2$, **then prove that :** $f(2) = f\left(\frac{1}{2}\right)$ *(Luxor 2014)*

Sheet (5)

Polynomial functions Part (2)

Sheet (5) Polynomial functions Part (2)

First The linear function

Definition

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX + b$ where $a \in \mathbb{R} - \{0\}$, $b \in \mathbb{R}$ is called a linear function (it is a polynomial function of the first degree).

Examples of linear functions :

- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = X - 1$
- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = 2X + 1$
- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = 3X$

Notice that :

- In each of the shown functions, the index of X is 1, therefore each of them is a function of the first degree.

The graphical representation of the linear function

- The linear function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX + b$, $a \in \mathbb{R} - \{0\}$, $b \in \mathbb{R}$ is represented graphically by a straight line intersecting :

  – The y-axis at the point $(0, b)$ – The X-axis at the point $\left(\frac{-b}{a}, 0\right)$

- To represent a linear function, it is enough to find two ordered pairs belonging to the function.
- You can find a third ordered pair to check that the three points are on the same straight line.

Example (1) **Graph each of the following linear functions :**

1 $f : f(X) = 2X - 3$ **2** $r : r(X) = -\frac{1}{2}X$

Solution

1 To graph this function :

- We determine 3 ordered pairs belonging to the function.

$\because f(X) = 2X - 3$

$\therefore f(-1) = 2(-1) - 3 = -5$ $\therefore (-1, -5) \in f$

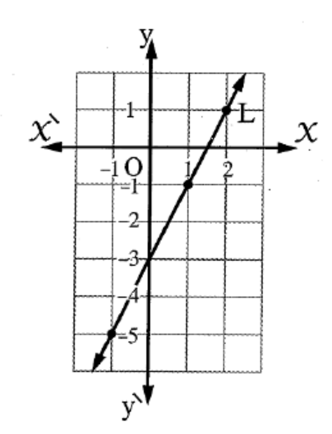
$, f(1) = 2 \times 1 - 3 = -1$ $\therefore (1, -1) \in f$

and $f(2) = 2 \times 2 - 3 = 1$ $\therefore (2, 1) \in f$

We can arrange these ordered pairs in the following table :

x	−1	1	2
$y = f(x)$	−5	−1	1

- We locate these three points which represents the three ordered pairs in the Cartesian plane and draw the straight line L which passes through any two points of them.
  Then we check that the third point lies on the same straight line.
  Then this straight line is the graphical representation of this function.



Notice that :

We can get the points of intersection of this straight line with the two axes and use them in representation.

- The point of intersection with y-axis = (0 , b) = (0 , − 3)
- The point of intersection with x-axis = $\left(-\frac{b}{a}, 0\right) = \left(\frac{3}{2}, 0\right)$

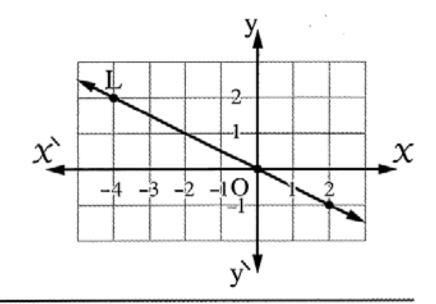
2 $\because r(x) = -\frac{1}{2}x$ $\therefore$

x	0	2	−4
$y = f(x)$	0	−1	2

Notice that :

If the coefficient of x is a fraction , it is better to choose numbers divisible by the denominator of this fraction to facilitate the representation.

From the opposite graph notice that , the straight line L passes through the origin point O (0 , 0)



Generally

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX$, $a \in \mathbb{R}^*$

is represented graphically by a straight line passing through the origin point (0 , 0)

Second The constant function

Definition

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = b$, $b \in \mathbb{R}$ is called a constant function.

For example:

$f : f(X) = 5$ is a constant function where

$f(1) = 5$, $f(0) = 5$, $f(-2) = 5$, ... and so on.

Graphical representation of the constant function

The constant function $f : f(X) = b$ (where $b \in \mathbb{R}$) is represented by a straight line parallel to X-axis and passes through the point (0 , b) this line is :

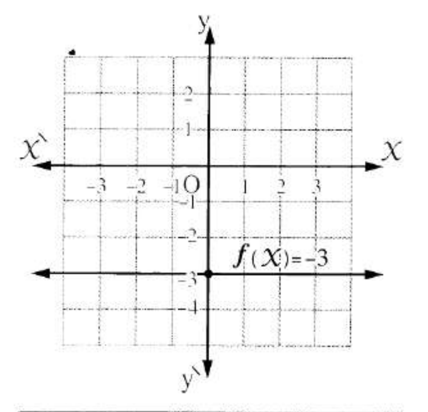
- above X-axis if $b > 0$
- below X-axis if $b < 0$
- coincident with X-axis if $b = 0$

The following examples illustrate that :

$f : f(X) = 2$

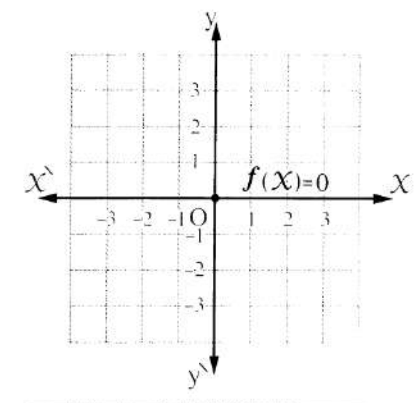
The straight line is above X-axis and passes through the point (0 , 2)

$f : f(X) = -3$



The straight line is below X-axis and passes through the point (0 , – 3)

$f : f(X) = 0$



The straight line is coincident with X-axis and passes through the point (0 , 0)

Third The quadratic function

Definition

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = ax^2 + bx + c$
where a , b and c are real numbers , $a \neq 0$
is called a quadratic function (it is a polynomial function of the second degree).

Examples of quadratic functions :

- $f : \mathbb{R} \longrightarrow \mathbb{R} , f(x) = x^2$
- $f : \mathbb{R} \longrightarrow \mathbb{R} , f(x) = x^2 - 2$
- $f : \mathbb{R} \longrightarrow \mathbb{R} , f(x) = 3x^2 - 7x + 2$
- $f : \mathbb{R} \longrightarrow \mathbb{R} , f(x) = 6 - x^2 + x$

Notice that :

In each of the shown functions , the highest index of x is 2 therefore each of them is a function of the 2^{nd} degree.

Example 2 **Graph each of the following quadratic functions :**

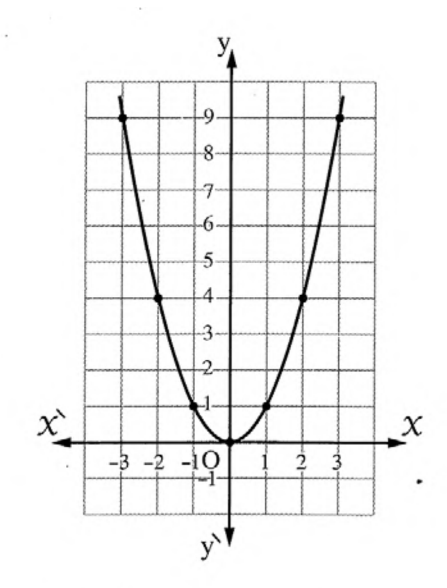
1 $f : f(x) = x^2$ taking $x \in [-3 , 3]$

2 $f : f(x) = -x^2$ taking $x \in [-3 , 3]$

Solution

1 $f(x) = x^2$

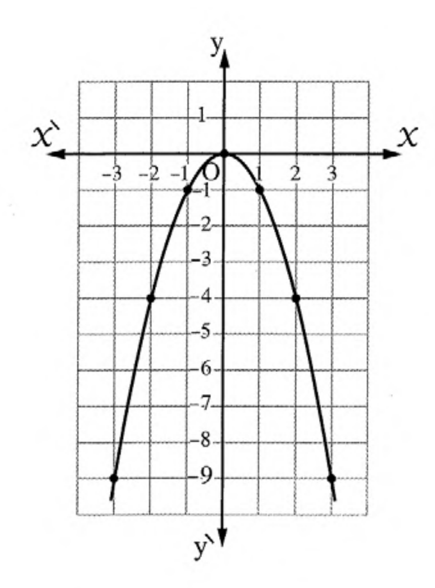
x	−3	−2	−1	0	1	2	3
$f(x)$	9	4	1	0	1	4	9



Notice that : The coefficient of $x^2 > 0$

2 $f(x) = -x^2$

x	−3	−2	−1	0	1	2	3
$f(x)$	−9	−4	−1	0	−1	−4	−9



Notice that : The coefficient of $x^2 < 0$

1 The curve is symmetric with respect to y-axis *i.e.* the y-axis is the line of symmetry of the curve and its equation is $x = 0$

2 The point (0 , 0) is the point of the vertex of the curve , it is considered as a minimum value point of the curve because the whole curve lies up on it.

3 The minimum value of the function is zero it equals the y-coordinate of the vertex of the curve.

1 The curve is symmetric with respect to y-axis *i.e.* the y-axis is the line of symmetry of the curve and its equation is $x = 0$

2 The point (0 , 0) is the point of the vertex of the curve , it is considered as a maximum value point of the curve because the whole curve lies below it.

3 The maximum value of the function is zero it equals the y-coordinate of the vertex of the curve.

Generally , for any quadratic function

1 If the coefficient of x^2 is positive , then the curve is open upwards and the function has a minimum value point.

2 If the coefficient of x^2 is negative , then the curve is open downwards and the function has a maximum value point.

Example (3) **Graph the function : $f : f(x) = x^2 - 2x - 3$ taking $x \in [-2 , 4]$ from the graph , find :**

1 The point of the vertex of the curve.

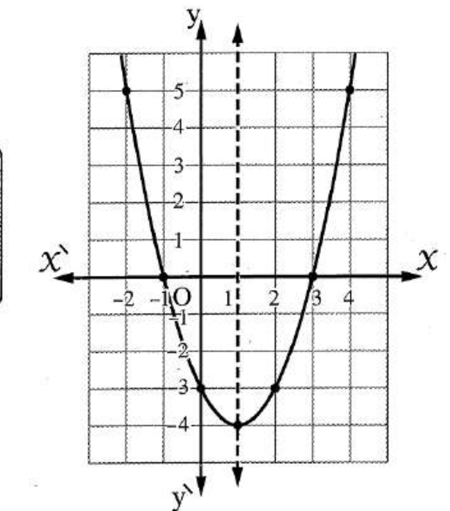
2 The equation of the line of symmetry.

3 The maximum or minimum value of the function.

Solution

$f(x) = x^2 - 2x - 3$

x	−2	−1	0	1	2	3	4
$f(x)$	5	0	−3	−4	−3	0	5



From the graph , we deduce that :

1 The vertex of the curve is (1 , −4)

2 The equation of the line of symmetry is $x = 1$, it is a straight line parallel to y-axis and passes through the

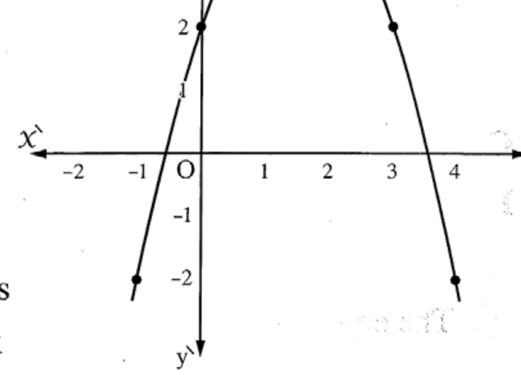
vertex of the curve.

3 The minimum value of the function = – 4

Example 4 **Graph the function $f : f(x) = -x^2 + 3x + 2$ taking $x \in [-1, 4]$ and from the graph , find :**

1 The maximum value or minimum value of the function.

2 The equation of the line of symmetry.

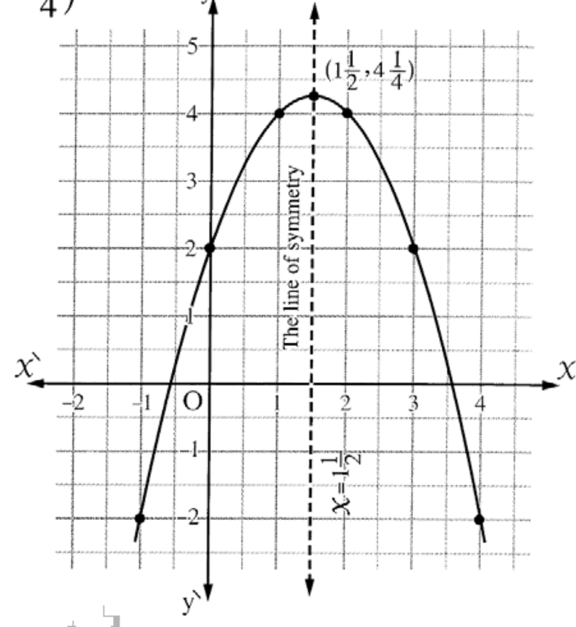

Illustrative graph

Solution $f(x) = -x^2 + 3x + 2$

x	–1	0	1	2	3	4
$f(x)$	–2	2	4	4	2	–2

When we represent these ordered pairs , we notice that the point of the vertex of the curve is not among these points which makes the drawing.
of the dotted part in the opposite figure is inaccurate , so the studying of the curve will be difficult , then we should find the vertex point of the curve algebraically as the following :

∴ The vertex of the curve is $\left(1\frac{1}{2}, 4\frac{1}{4}\right)$



From the graph , we find that :

1 The maximum value of the function = $4\frac{1}{4}$

2 The equation of the line of symmetry is $x = 1\frac{1}{2}$
it is a straight line parallel to y-axis and passes through the vertex of the curve.

Finding the point of the vertex of the curve :

At the point of the vertex of the curve of the quadratic function , it will be :

- The X-coordinate $= \frac{-b}{2a}$
- The y-coordinate $= f\left(\frac{-b}{2a}\right)$

where b is the coefficient of X , a is the coefficient of X^2

$\therefore$ X at the vertex of the curve $= \frac{-3}{2 \times -1} = \frac{-3}{-2} = 1\frac{1}{2}$

, $\because f\left(1\frac{1}{2}\right) = -\frac{9}{4} + \frac{9}{2} + 2 = 4\frac{1}{4}$

Choose the correct answer :

1 If : $f(X) = 3$, then $f(2) =$ ....

(a) 2 (b) 3 (c) 9 (d) 6

2 If : $f(X) = 5$, then $f(3) =$ .... *(Beni Suef 2013)*

(a) 5 (b) 15 (c) 8 (d) $\frac{3}{5}$

3 If $f(X) = 7$, then $f(3) =$ .... (Souhag 2011)

(a) 10 (b) 3 (c) 7 (d) 5

4 If $f(X) = 5$, then $f(3) - f(1) =$ .... (Cairo 2006)

(a) $f(2)$ (b) 2 (c) zero (d) 10

5 If : $f(X) = 2$, then $f(3) - f(1) =$ .... *(Dakahlia 2013)*

(a) $f(2)$ (b) 2 (c) zero (d) 10

6 If : $f(X) = 2$, then $f(1) + f(-1) =$ ....

(a) zero (b) 1 (c) 2 (d) 4

7 If $f(X) = 3$, then $\frac{2f(3)}{3f(2)} =$ .... (Alex. 2005)

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 1 (d) $\frac{32}{23}$

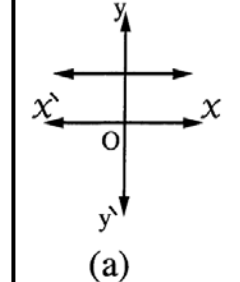
8 If : $f(2X) = 4$, then $f(-X) =$ .... *(Dakahlia 2009)*

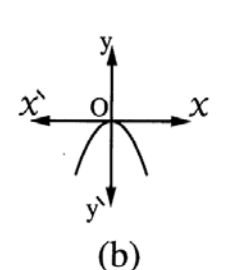
(a) -2 (b) -4 (c) 4 (d) 2

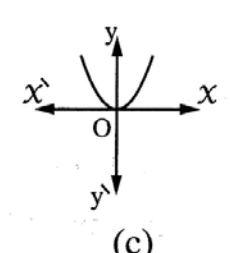
9 If the curve of the function f where $f(X) = X^2 - a$ passes through the point $(1, 0)$, then a = ... (Alex. 2011)

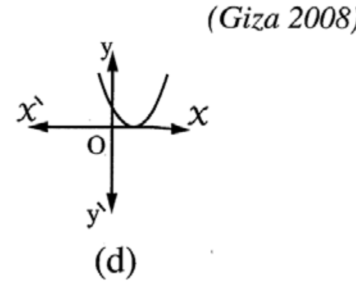
(a) ± 1 (b) -1 (c) 1 (d) zero

10	If the function f where $f(X) = 5X + 4$ is represented by a straight line passing through the point (3 , b), then b equals …………… (a) 5 (b) 4 (c) 3 (d) 19
11	If the point (–3 , y) is located on the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$, where $f(X) = 2X + 7$, then y = …………… (a) 1 (b) 3 (c) 5 (d) 7
12	The function $f : f(X) = 5X$ is represented graphically by a straight line passing through the point …………… (a) (5 , 5) (b) (0 , 0) (c) (5 , 0) (d) (0 , 5)
13	10 The graph of the function $f : f(X) = X^2 - 2X + 1$ is the graph number ………. *(Giza 2008)*


(a)


(b)


(c)


(d)

Essay problems:

1	If : $f(X) = X - 6$ and $\frac{1}{3} f(a) = -2$, then find the value of a
2	If : $f(X) = 2X + b$, $h(X) = b$ where f and h are polynomials functions and if $f(1) + h(4) = 12$, then find : $f(4) + h(-1)$ « 18 » *(El-Sharkia 2013)*
3	If : $f(X) = 4X + b$, $f(3) = 15$, then find the value of : b
4	If : $f(X) = X^2 - X + 3$, find : $f(-2)$, $f(\text{zero})$, $f(\sqrt{3})$
5	Represent graphically the function $f : \mathbb{R} \longrightarrow \mathbb{R}$, where $f(X) = 2 - X$ and find the points of intersection of the straight line by the two coordinate axes.

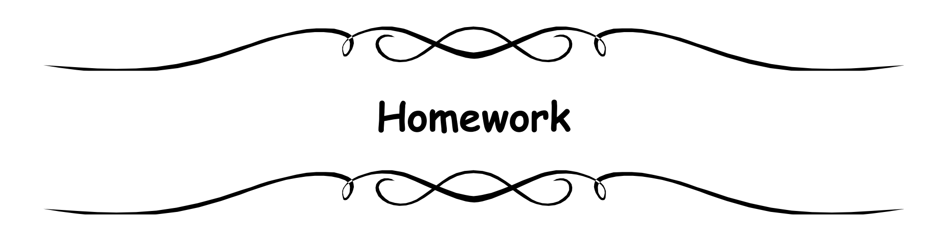
6 Represent graphically the function f where $f(\mathcal{X}) = \mathcal{X}^2$, $\mathcal{X} \in \mathbb{R}$ consider $\mathcal{X} \in [-3 , 3]$ and from the drawing deduce the coordinate of the vertex of the curve, and the equation of the symmetry axis and the minimum or the maximum value of the function.

7 Draw the curve of the function f such that $f(\mathcal{X}) = \mathcal{X}^2 - 3$ taking that $\mathcal{X} \in [-3 , 3]$, and **from the graph deduce :**

(1) The coordinates of the vertex of the curve.

(2) The maximum or minimum value of the curve.

(3) The equation of the symmetric axis.



Homework

Complete each of the following :

1 If : $f(\mathcal{X}) = 5$, then $f(3) = \dots\dots\dots$ *(Beni Suef 2013)*

(a) 5 (b) 15 (c) 8 (d) $\frac{3}{5}$

2 If $f(\mathcal{X}) = 5$, then $f(3) - f(1) = \dots\dots\dots$ (Cairo 2006)

(a) $f(2)$ (b) 2 (c) zero (d) 10

3 If : $f(\mathcal{X}) = 2$, then $f(3) - f(1) = \dots\dots\dots$ *(Dakahlia 2013)*

(a) $f(2)$ (b) 2 (c) zero (d) 10

4) If : $f(\mathcal{X}) = 2$, then $f(1) + f(-1) = \dots\dots\dots$

(a) zero (b) 1 (c) 2 (d) 4

5 If $f(\mathcal{X}) = 3$, then $\frac{2f(3)}{3f(2)} = \dots\dots\dots$ (Alex. 2005)

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 1 (d) $\frac{32}{23}$

6 If : $f(2\mathcal{X}) = 4$, then $f(-\mathcal{X}) = \dots\dots\dots$ *(Dakahlia 2009)*

(a) -2 (b) -4 (c) 4 (d) 2

7 If the point (–3 , y) is located on the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$, where $f(X) = 2X + 7$, then y = ...

(a) 1 (b) 3 (c) 5 (d) 7

Essay problems:

1 If $f : \mathbb{R} \longrightarrow \mathbb{R}$ is represented by a straight line cuts y-axis at (b , 3) where $f(X) = 6X - a$

Find the value of : 2 a + 7 b

2 If the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 2X - a$ cuts y-axis at the point (b , 5) **Find the value of :** 3 a + 2 b

3 Graph the function f where

📖 $f : f(X) = X^2 - 2$ taking $X \in [-3 , 3]$ *(Giza , Qena 2013)*

, from the drawing find :

(1) The coordinates of the vertex of the curve.

(2) The equation of the axis of symmetry.

(3) The minimum value of the function.

4 Draw the graph of the function f , where $f(X) = X^2 - 2X - 3$, $X \in [-2 , 4]$

From the graph find :

(1) The coordinates of vertex of the curve. (2) The equation of the line of symmetry

(3) The minimum or the maximum value of the function.

5 Graph the function f where

📖 $f : f(X) = 2 - X^2$ taking $X \in [-3 , 3]$ *(Assiut 2013 , El-Gharbia 2014)*

, from the drawing find :

(1) The coordinates of the vertex of the curve.

(2) The equation of the axis of symmetry.

(3) The minimum value of the function.

6 **Represent graphically the linear function $f(X) = 2X + 1$ and find the points of intersection of the straight line representing it, with the two coordinate axes.**

7 If the curve of the function : $f(X) = aX - 5$ passes through the point (2 , 3) , then find the value of a and find the point of intersection of the straight line which represents it with y-axis.

8 If the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$, where $f(X) = 6X - a$ cuts the y-axis at the point (b , 3) , **then find the value of :** b and a

9 Graph the function f where $f(X) = 2X - X^2$ in the interval $[-1 , 3]$, **from the graph determine :**

(1) The coordinates of the vertex of the curve.

(2) The equation of the axis of symmetry.

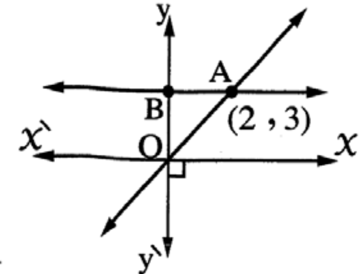
10 Represent graphically the function f where $f(X) = 4 - X^2$ in $[-3 , 3]$ **and from the graph find :**

(1) The coordinates of the vertex of the curve

(2) The equation of the symmetry axis.

(3) The maximum value or the minimum value.

11 **In the opposite figure :**



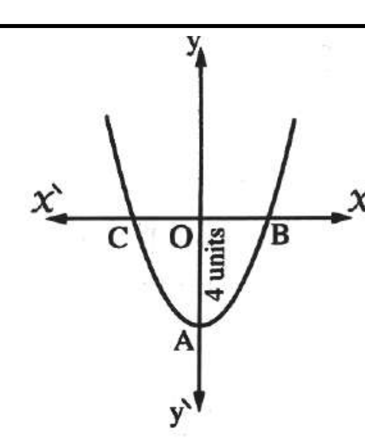
The constant function f is represent graphically by straight line $\overleftrightarrow{BA}$ and the linear function g is represent graphically by straight line $\overleftrightarrow{OA}$ where A = (2 , 3)

1 Write the rule of the function f and the rule of the function g

2 Find the value of : $f(-10) + g(6)$

(El-Sharkia 2014)

12 **The opposite figure :**



represents the curve of the function f , where :

$f(X) = X^2 - m$ If OA = 4 units , **find :**

(1) The value of m

(2) The area of the triangle with vertices A , B and C

Sheet (6)
Ratio and Proportion

Sheet (6) Ratio and Proportion

First : The Ratio : -

Generally

If a and b are two real numbers , then :
The ratio between a and b is written a : b or $\frac{a}{b}$ and is read a to b where :
a is called the antecedent of the ratio , b is called the consequent of the ratio, a and b are called together the two terms of the ratio.

Properties of the ratio

1 The value of the ratio does not change if each of its terms is multiplied or divided by the same non-zero real number.

2 The value of the ratio ($\neq 1$) changes if we add or subtract (to or from) each of its two terms a non-zero real number.

First : The Proportion: -

Definition of proportion

It is the equality of two ratios or more.

i.e. If $\frac{a}{b} = \frac{c}{d}$, then the quantities a , b , c and d are proportional.
And vice versa : If a , b , c and d are proportional , then : $\frac{a}{b} = \frac{c}{d}$

- a is called the first proportional.
- b is called the second proportional.
- c is called the third proportional.
- d is called the fourth proportional.

a and d are called extremes and b and c are called means.

For example:

The numbers 1 , 4 , 7 and 28 are proportional numbers , because $\frac{1}{4} = \frac{7}{28}$
And : 1 is the first proportional, 4 is the second proportional , 7 is the third proportional , 28 is the fourth proportional, 1 and 28 are the extremes of this proportion and 4 and 7 are the means.

Properties of proportion

Property (1)

If $\frac{a}{b} = \frac{c}{d}$, then : $a \times d = b \times c$ (The product of the extremes = the product of the means)

Property (2)

If $a \times d = b \times c$, then $\frac{a}{b} = \frac{c}{d}$

Also we can deduce that :

- If $a \times d = b \times c$, then $\frac{a}{c} = \frac{b}{d}$
- If $a \times d = b \times c$, then $\frac{b}{a} = \frac{d}{c}$
- If $a \times d = b \times c$, then $\frac{c}{a} = \frac{d}{b}$

Property (3)

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$

i.e. $\frac{\text{The antecedent of the first ratio}}{\text{The antecedent of the second ratio}} = \frac{\text{The consequent of the first ratio}}{\text{The consequent of the second ratio}}$

For example:

If $\frac{a}{4} = \frac{b}{3}$, then $\frac{a}{b} = \frac{4}{3}$ and $\frac{b}{a} = \frac{3}{4}$

Property (4)

If $\frac{a}{b} = \frac{c}{d}$, then $a = cm$ and $b = dm$ (where m is a constant $\neq 0$)

For example:

If $\frac{a}{b} = \frac{3}{4}$, then : $a = 3\ m$, $b = 4\ m$ (where m is a constant $\neq 0$)

Important remark

If a , b , c and d are proportional quantities and we assume that : $\frac{a}{b} = \frac{c}{d} = m$, then

ⓐ $= bm$, ⓒ $= dm$

For example:

If $\frac{a}{b} = \frac{c}{d} = \frac{3}{4}$, then $a = \frac{3}{4}\ b$, $c = \frac{3}{4}\ d$

Generally

If a , b , c , d , e , f , ... are proportional quantities and we assume that :

$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \cdots = m$, then ⓐ $= bm$, ⓒ $= dm$, ⓔ $= fm$, ⋯

Property (5)

If we consider the proportion : $\frac{9}{15} = \frac{6}{10} = \frac{3}{5}$

- If we add the antecedents and consequents of the 1st and the 2nd ratios, we get the ratio $\frac{9+6}{15+10} = \frac{15}{25} = \frac{3}{5}$ which is one of given ratios.
- Also if we add the antecedents and consequents of the 2nd and the 3rd ratios, we get the ratio $\frac{6+3}{10+5} = \frac{9}{15} = \frac{3}{5}$ = one of the given ratios.
- If we add the antecedents and consequents of the three given ratios, we get the ratio $\frac{9+6+3}{15+10+5} = \frac{18}{30} = \frac{3}{5}$ = one of the given ratios.

i.e. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \cdots$ and m_1 , m_2 , m_3 , ⋯ are non-zero real numbers

, then $\frac{m_1 a + m_2 c + m_3 e + \cdots}{m_1 b + m_2 d + m_3 f + \cdots}$ = one of the given ratios

Choose the correct answer :

1 If : 24 , X , 6 and 3 are proportional quantities , then X = ...

A) 9 B) 12 C) 18 D) 48

2 The fourth proportional for the 2 , 6 , 9 is ....

A) 12 B) 18 C) 27 D) 54

3 The fourth proportional for the 3 , 6 , 6 is........................

A) – 12 B) 6 C) 9 D) 12

4 If $\frac{a}{b} = \frac{3}{2}$, then $\frac{a+b}{a-b}$ = ...

A) $\frac{3}{2}$ B) 5 C) $\frac{4}{5}$ D) 2

5	If : $\frac{a}{b} = \frac{3}{4}$, then $4a - 3b + 5 =$ .... A) 0 B) 1 C) 3 D) 5
6	If : $\frac{a}{b} = \frac{5}{3}$, then $\frac{3a}{5b} =$ .... A) 1 B) $\frac{5}{3}$ C) 3 D) 5
7	If : $\frac{a}{b} = \frac{c}{d} = \frac{3}{4}$, then $\frac{a+c}{b+d} =$ .... A) $\frac{3}{4}$ B) $\frac{7}{4}$ C) $\frac{3}{7}$ D) $\frac{9}{16}$
8	If : $\frac{a}{2} = \frac{b}{3}$, then $\frac{b-a}{b+a} =$ .... A) $\frac{1}{5}$ B) $\frac{1}{3}$ C) $\frac{2}{5}$ D) $\frac{3}{5}$
9	If : $\frac{a}{12} = \frac{b}{5} = \frac{a-2b}{k}$, then $k =$ .... A) 1 B) 2 C) 3 D) 4
10	If : $\frac{a}{5} = \frac{b}{4} = \frac{a+b}{k}$, then $k =$ .... A) 5 B) 4 C) 9 D) 1
11	If : $\frac{X}{Y} = \frac{Z}{1}$ which of the following is right .... A) $\frac{X}{1} = \frac{Y}{Z}$ B) $\frac{X}{Z} = \frac{1}{Y}$ C) $\frac{X}{Y} = \frac{1}{Z}$ D) $\frac{X}{Z} = \frac{Y}{1}$
12	If : $\frac{X}{2} = \frac{Y}{7} = \frac{X+Y}{K}$, then $K =$ .... A) 4 B) 10 C) 6 D) 14
13	$4X = 25Y$, then $\frac{X}{Y} =$ .... A) $\frac{4}{25}$ B) $\frac{2}{5}$ C) $\frac{5}{2}$ D) $\frac{25}{4}$
14	If : A , Y , B and 2 X are proportional, then : $\frac{A}{B} =$ .... A) 2 : 1 B) 1 : 2 C) 1 : 3 D) 1 : 4

15 If : $\frac{3a}{5b} = \frac{1}{2}$, then : $\frac{a}{b}$ = ……………………

A) $\frac{6}{5}$ B) $\frac{5}{6}$ C) $\frac{2}{3}$ D) $\frac{3}{2}$

16 If : $\frac{a+b}{5} = \frac{a-b}{3}$, then : $\frac{a}{b}$ = ……………………

A) $\frac{1}{2}$ B) 2 C) 4 D) $\frac{1}{4}$

17 If : 2 a = 3 b , then $\frac{5b}{a}$ = ……………………

A) $\frac{5}{3}$ B) $\frac{5}{2}$ C) $\frac{15}{2}$ D) $\frac{10}{3}$

18 If 3 x = 5 y , then $\frac{5y}{3x}$ = ……………………

A) 1 B) 2 C) $\frac{3}{5}$ D) $\frac{5}{3}$

19 If : 4 x = 5 y, then : $\frac{5y}{4x}$ = ……………………

A) 1 B) 2 C) 3 D) 4

Essay problems:

1 If $\frac{x-2y}{x+3y} = \frac{1}{3}$, find : $\frac{y}{x}$

2 If $\frac{a}{b} = \frac{3}{4}$, then find the value of $\frac{4a+b}{2a-b}$

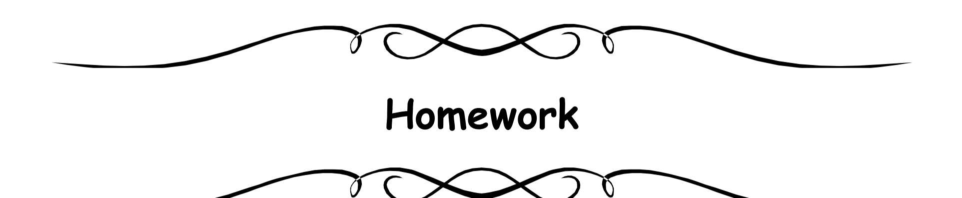
3 If $\frac{a}{b} = \frac{3}{5}$, then find the value of 7 a + 9 b : 4 a + 2 b

4 If $\frac{21x+a}{7x+b} = \frac{a}{b}$, where $x \neq 0$ then find the value of : $\frac{a+2b}{2a}$

5 Prove that : a , b , c and d are proportional quantities if : $\frac{a+b}{b} = \frac{c+d}{d}$

6 If : $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$, then prove that : $3X^2 + 3y^2 + z^2 = (2X + y)^2$

7	Find the number which if it is added to the two terms of the ratio 7:11 , it will be 2 : 3
8	The ratio between two integers is $\frac{3}{4}$, if we add 4 to the small number and subtract 3 from the great number , the ratio will become 8 : 9 Find the two numbers.
9	Two integers , the ratio between them is 2 : 3 , if you add to the first 7 and subtract from the second 12 , the ratio between them becomes 5 : 3 Find the two numbers.



Homework

Choose the correct answer :

1	The first proportional of the numbers : 21 , 15 and 35 is .... A) 9 B) 3 C) 7 D) $\frac{7}{3}$
2	The fourth proportional for the 8 , 6 and 4 is.......................... A) 2 B) 3 C) 4 D) 7
3	The fourth proportional for the 9 , 12 , 3 is .... A) 6 B) 4 C) 5 D) 1
4	If $\frac{a}{b} = \frac{3}{2}$, then $\frac{a-b}{a+b}$ = ... A) $\frac{3}{2}$ B) 5 C) $\frac{4}{5}$ D) 2
5	If : $\frac{a}{b} = \frac{3}{7}$, then $\frac{a}{b-a}$ =.......................... A) $\frac{3}{4}$ B) $\frac{3}{7}$ C) $\frac{3}{10}$ D) Otherwise
6	If : $\frac{a}{b} = \frac{5}{4}$, then $\frac{a+b}{a-b}$ = ... A) $\frac{5}{4}$ B) 9 C) $\frac{4}{5}$ D) 2

7 If : $\frac{a}{5} = \frac{b}{7}$, then $7a - 5b + 3 =$ ....

A) 3 B) 7 C) 5 D) 2

8 If : $\frac{a}{5} = \frac{b}{2} = \frac{a - 2b}{k}$, then k = ...

A) 5 B) 2 C) 3 D) 1

9 If : $\frac{a}{2} = \frac{b}{3} = \frac{4a - 2b}{c}$, then c = ...

A) -2 B) 2 C) $-\frac{1}{2}$ D) $\frac{1}{2}$

10 If : $\frac{X}{3} = \frac{8}{12}$, then X = ...

A) 6 B) 5 C) 4 D) 2

11 If : $\frac{X}{2} = \frac{Y}{7} = \frac{2X + Y}{A}$, then A = ...

A) 9 B) 11 C) 16 D) 5

12 If : $\frac{X}{5} = \frac{Y}{3} = \frac{X - Y}{A}$, then A = ...

A) -2 B) 2 C) 8 D) 15

13 If : X , Y , 2 and 3 are proportional, then : $\frac{Y}{X} =$ ....

A) $\frac{3}{2}$ B) $\frac{2}{3}$ C) 3 D) 2

14 If : 5 a , 2 , 3 b , 7 are four proportional quantities , then : $\frac{a}{b} =$ ....

A) $\frac{3}{7}$ B) $\frac{6}{35}$ C) $\frac{3}{5}$ D) $\frac{3}{2}$

15 If : $\frac{a + 2b}{a - b} = \frac{2}{3}$, then : $\frac{b}{a} =$ ....

A) $\frac{1}{8}$ B) 8 C) $-\frac{1}{8}$ D) -8

16 If : $4X^2 + 9Y^2 = 12XY$, then : $\frac{X}{Y} =$ ....

A) $\frac{3}{2}$ B) $\frac{2}{3}$ C) $-\frac{2}{3}$ D) $-\frac{3}{2}$

17

If : $2x = 7y$, then $(\frac{x}{y})^{-1} =$ ....

A) $\frac{2}{7}$ B) $\frac{7}{2}$ C) $\frac{49}{4}$ D) $\frac{4}{49}$

18

If $3a = 8b$, then : $\frac{2a}{b} =$ ....

A) 24 B) 16 C) $\frac{16}{3}$ D) $\frac{3}{8}$

19

The ratio between the area of a square shaped region of side length L to the area of another square shaped region of side length 2 L is..............

A) 1 : 2 B) L : 4 C) 1 : 4 D) 4 : 1

21

If : 24 , X , 6 and 3 are proportional quantities , then X = ...

A) 9 B) 12 C) 18 D) 48

22

The fourth proportional for the 2 , 6 , 9 is ....

A) B) C) D)

23

The fourth proportional for the 8 , 6 and 4 is..........................

A) B) C) D)

24

If $\frac{a}{b} = \frac{3}{2}$, then $\frac{a+b}{a-b} =$ ....

A) $\frac{3}{2}$ B) 5 C) $\frac{4}{5}$ D) 2

25

If : $\frac{a}{b} = \frac{3}{4}$, then $4a - 3b + 5 =$..........................

A) 0 B) 1 C) 3 D) 5

26

If : $\frac{a}{b} = \frac{5}{3}$, then $\frac{3a}{5b} =$ ....

A) 1 B) $\frac{5}{3}$ C) 3 D) 5

27

If : $\frac{a}{b} = \frac{c}{d} = \frac{3}{4}$, then $\frac{a+c}{b+d} =$ ....

A) $\frac{3}{4}$ B) $\frac{7}{4}$ C) $\frac{3}{7}$ D) $\frac{9}{16}$

28

If : $\frac{a}{2} = \frac{b}{3}$, then $\frac{b-a}{b+a} =$ ....

A) $\frac{1}{5}$ B) $\frac{1}{3}$ C) $\frac{2}{5}$ D) $\frac{3}{5}$

29	If : $\frac{a}{12} = \frac{b}{5} = \frac{a-2b}{k}$, then k = ... A) 1 B) 2 C) 3 D) 4
30	If : $\frac{a}{5} = \frac{b}{4} = \frac{a+b}{k}$, then k = ... A) 5 B) 4 C) 9 D) 1
31	If : $\frac{X}{Y} = \frac{Z}{1}$ which of the following is right .... A) $\frac{X}{1} = \frac{Y}{Z}$ B) $\frac{X}{Z} = \frac{1}{Y}$ C) $\frac{X}{Y} = \frac{1}{Z}$ D) $\frac{X}{Z} = \frac{Y}{1}$

Essay problems:

1	If $\frac{x-2y}{x+3y} = \frac{1}{3}$, find : $\frac{y}{x}$
2	If $\frac{a}{b} = \frac{3}{4}$, then find the value of $\frac{4a+b}{2a-b}$
3	If $\frac{x}{y} = \frac{2}{3}$, then find the value of ratio : $\frac{3x+2y}{6y-x}$
4	If $\frac{a}{b} = \frac{3}{5}$, then find the value of 7 a + 9 b : 4 a + 2 b
5	If $\frac{21x+a}{7x+b} = \frac{a}{b}$, where $x \neq 0$ then find the value of : $\frac{a+2b}{2a}$
6	Prove that : a , b , c and d are proportional quantities if : $\frac{a+b}{b} = \frac{c+d}{d}$
7	Prove that : a , b , c and d are proportional quantities if : $\frac{a}{b-a} = \frac{c}{d-c}$
8	If : $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$, then prove that : $\frac{2y-z}{3x-2y+z} = \frac{1}{2}$
9	If : $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$, then prove that : $3X^2 + 3y^2 + z^2 = (2X + y)^2$

10	Find the number which if it is added to the two terms of the ratio 7:11 , it will be 2 : 3
11	Find the number that if we subtract thrice of it from each of the two terms of the Ratio $\frac{49}{69}$, the ratio becomes $\frac{2}{3}$
12	Find the number which if its square is added to each of the two terms of ratio 7 : 11 it becomes 4 : 5
13	Two integers, the ratio between them is 3 : 7 and if we subtracted 5 from each term , the ratio between each of them becomes 1 : 3, find the two numbers.
14	The ratio between two integers is $\frac{3}{4}$, if we add 4 to the small number and subtract 3 from the great number , the ratio will become 8 : 9 Find the two numbers.
15	Two integers , the ratio between them is 2 : 3 , if you add to the first 7 and subtract from the second 12 , the ratio between them becomes 5 : 3 Find the two numbers.
16	In the opposite figure: Alaa shaded $\frac{5}{6}$ the area of the circle, $\frac{2}{3}$ the area of the triangle, find the ratio between the area of the circle and the area of the triangle.

Sheet (7)

Continued Proportion

Sheet (7) Continued Proportion

Definition :

The quantities a , b and c are said to be in continued proportion if $\frac{a}{b} = \frac{b}{c}$

In this proportion , **a** is called the **first proportion** , **c** is called the **third proportion** and **b** is called the **middle proportion (proportional mean)**.

For Example : -

The numbers 4 , 6 and 9 form a continued proportion because : $\frac{4}{6} = \frac{6}{9}$ or because : $(6)^2 = 4 \times 9$ where 6 is the middle proportion , 4 is the first proportion and 9 is the third proportion.

Notice That : -

1 If a , b and c are in continued proportion , then : $b^2 = a\,c$ *i.e.* $b = \pm\sqrt{ac}$ and the two quantities a and c should be either both positive or both negative.

2 For any two positive numbers or any two negative numbers x and y , there are two middle proportions $\left(\sqrt{xy} \text{ and } -\sqrt{xy}\right)$

Remark : -

If a , b and c are in continued proportion and we assume that : $\frac{a}{b} = \frac{b}{c} = m$

, then $\frac{b}{c} = m$ $\therefore$ $b = cm$ (1)

, $\because \frac{a}{b} = m$ $\therefore a = bm$

Substituting for b from (1) : $\therefore a = (cm)\,m$ $\therefore a = cm^2$

i.e. If $\frac{a}{b} = \frac{b}{c} = m$, then $\begin{cases} b = cm \\ a = cm^2 \end{cases}$

General Definition : -

The quantities a , b , c , d , ⋯ are in continued proportion if : $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \cdots$

For Example : -

The numbers 16 , 24 , 36 and 54 are in continued proportion

because : $\frac{16}{24} = \frac{24}{36} = \frac{36}{54}$, each ratio = $\frac{2}{3}$

Remark : -

If a , b , c and d are in continued proportion and we assume that : $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$, then :

$\frac{c}{d} = m$ $\therefore$ (c) $= dm$ (1)

, $\frac{b}{c} = m$ $\therefore b = cm$

Substituting for c from (1) : $\therefore b = (dm)\, m$ $\therefore$ (b) $= dm^2$ (2)

, $\frac{a}{b} = m$ $\therefore a = bm$

Substituting for b from (2) : $\therefore a = (dm^2)\, m$ $\therefore$ (a) $= dm^3$

i.e. If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$, then $c = dm$, $b = dm^2$ and $a = dm^3$

Choose the correct answer :

1	The third proportion of the two numbers 9 and -12 is .... A) -16 B) 8 C) 16 D) 108
2	The third proportion of the two numbers 3 and 12 is .... A) $\frac{1}{2}$ B) 2 C) 9 D) 12
3	The middle proportion of the two numbers 3 and 27 is .... A) -9 B) 9 C) ± 9 D) 21
4	The middle proportion of the two numbers 4 and 36 is .... A) 32 B) 40 C) 12 D) ± 12
5	The third proportion of the two numbers 4 and 25 equals .... A) 10 B) 29 C) 100 D) ± 10
6	The proportional mean between a and b is .... A) a b B) $\sqrt{ab}$ C) $-\sqrt{ab}$ D) $\pm\sqrt{ab}$
7	The proportional mean between (X – 2) and (X + 2) is .... A) $\sqrt{X+2}$ B) $X^2 - 4$ C) $\pm\sqrt{X^2-4}$ D) $\sqrt{X^2-4}$

8	If the number 9 is the proportional mean of the two numbers 3 and k , then k = ... A) 18 B) 81 C) 27 D) 9
9	If the number 6 is the positive proportional mean of two numbers 2 and m , then m = ... A) 8 B) 12 C) 18 D) 36
10	If he middle proportion between 9 , k is 6 , then k = ... A) 6 B) 4 C) 9 D) 15
11	The number which is added to each of the numbers 1 , 3 , and 6 to become in continued proportion is .... A) 1 B) 2 C) 3 D) 6
12	The number which is added to each of the numbers 1 , 3 , 7 , 15 to be in continued proportion is .... A) 1 B) 2 C) 3 D) 4
13	If a , 2 , 4 , b are in continued proportion , then : a + b = ... A) 8 B) 1 C) 9 D) 7
14	If : $\frac{a}{b} = \frac{b}{c} = \frac{c}{5} = 2$, then a = ... A) 5×2^2 B) 40 C) 10 D) 2×5^3
15	If : $\frac{a}{b} = \frac{b}{c} = \frac{c}{5} = 2$, then $\frac{a}{d} =$ A) 2 B) 4 C) 8 D) 16
16	The positive middle proportion between 3 and 27 is .... A) 3 B) 4 C) 8 D) 9
17	If 2 , 6 , X + 15 are proportional , then X = ... A) 1 B) 2 C) 3 D) 4
18	If : 24 , X , 6 and 3 are proportional quantities , then X = ... A) 9 B) 12 C) 18 D) 48
19	If $\frac{a}{b} = \frac{3}{2}$, then $\frac{a+b}{a-b} =$ .... A) $\frac{3}{2}$ B) 5 C) $\frac{4}{5}$ D) 2

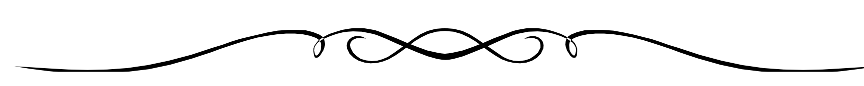
20	If : $\frac{a}{b} = \frac{3}{4}$, then $4a - 3b + 5 =$ .... A) 0 B) 1 C) 3 D) 5
21	If $3a = 8b$, then : $\frac{2a}{b}$ = ... A) 24 B) 16 C) $\frac{16}{3}$ D) $\frac{3}{8}$
22	The ratio between the area of a square shaped region of side length L to the area of another square shaped region of side length 2 L is.............. A) 1 : 2 B) L : 4 C) 1 : 4 D) 4 : 1
23	If : 24 , X , 6 and 3 are proportional quantities , then X = ... A) 9 B) 12 C) 18 D) 48
24	The fourth proportional for the 2 , 6 , 9 is .... A) B) C) D)

Essay problems:

1	**Find the middle proportion between 3 and 27**
2	**Find the third proportion for 6 and 12**
3	**If b is the middle proportion between a and c , prove that :** $\frac{a^2+b^2}{b^2+c^2} = \frac{a}{c}$
4	**If b is middle proportion between a and c , prove that :** $\frac{2c^2 - 3b^2}{c^2} = \frac{2b^2 - 3a^2}{b^2}$
5	**If a , b , c and d are in continued proportion prove that :** $\frac{c^2-d^2}{a-c} = \frac{bd}{a}$
6	**If a , b , c and d are in continued proportion prove that :** $\frac{a^2-3c^2}{b^2-3d^2} = \frac{b}{d}$
7	**If a , b , c and d are in continued proportion prove that :** $\frac{ab-cd}{b^2-c^2} = \frac{a+c}{b}$



Homework



Choose the correct answer :

1	The third proportion of the two numbers 3 and 12 is .... A) $\frac{1}{2}$ B) 2 C) 9 D) 12
2	The proportional mean between a and b is .... A) a b B) $\sqrt{a\,b}$ C) $-\sqrt{a\,b}$ D) $\pm\sqrt{a\,b}$
3	If he middle proportion between 9 , k is 6 , then k = ... A) 6 B) 4 C) 9 D) 15
4	The number which is added to each of the numbers 1 , 3 , and 6 to become in continued proportion is .... A) 1 B) 2 C) 3 D) 6
5	If : $\frac{a}{b} = \frac{b}{c} = \frac{c}{5} = 2$, then $\frac{a}{d} =$ A) 2 B) 4 C) 8 D) 16
6	If $\frac{a}{b} = \frac{3}{2}$, then $\frac{a+b}{a-b} =$ .... A) $\frac{3}{2}$ B) 5 C) $\frac{4}{5}$ D) 2
7	If : $\frac{a}{12} = \frac{b}{5} = \frac{a-2b}{k}$, then k =.......................... A) 1 B) 2 C) 3 D) 4
8	If : $\frac{a}{5} = \frac{b}{4} = \frac{a+b}{k}$, then k = ... A) 5 B) 4 C) 9 D) 1
9	The middle proportion of the two numbers 3 and 27 is .... A) -9 B) 9 C) ± 9 D) 21
10	The proportional mean between (X – 2) and (X + 2) is .... A) $\sqrt{X+2}$ B) $X^2 - 4$ C) $\pm\sqrt{X^2-4}$ D) $\sqrt{X^2-4}$
11	The number which is added to each of the numbers 1 , 3 , 7 , 15 to be in continued proportion is .... A) 1 B) 2 C) 3 D) 4

No.	Question
12	The positive middle proportion between 3 and 27 is .... A) 3 B) 4 C) 8 D) 9
13	If : $\frac{a}{b} = \frac{3}{4}$, then 4 a – 3 b + 5 =.......................... A) 0 B) 1 C) 3 D) 5
14	If : $\frac{X}{Y} = \frac{Z}{1}$ which of the following is right .... A) $\frac{X}{1} = \frac{Y}{Z}$ B) $\frac{X}{Z} = \frac{1}{Y}$ C) $\frac{X}{Y} = \frac{1}{Z}$ D) $\frac{X}{Z} = \frac{Y}{1}$
15	The middle proportion of the two numbers 4 and 36 is .... A) 32 B) 40 C) 12 D) ± 12
16	If the number 9 is the proportional mean of the two numbers 3 and k , then k = ... A) 18 B) 81 C) 27 D) 9
17	If a , 2 , 4 , b are in continued proportion , then : a + b = ... A) 8 B) 1 C) 9 D) 7
18	If 2 , 6 , X + 15 are proportional , then X = ... A) 1 B) 2 C) 3 D) 4
19	If : $\frac{a}{b} = \frac{c}{d} = \frac{3}{4}$, then $\frac{a+c}{b+d}$ = ... A) $\frac{3}{4}$ B) $\frac{7}{4}$ C) $\frac{3}{7}$ D) $\frac{9}{16}$
20	If : $\frac{X}{2} = \frac{Y}{7} = \frac{X+Y}{K}$, then K =.......................... A) 4 B) 10 C) 6 D) 14
21	The third proportion of the two numbers 9 and – 12 is .... A) – 16 B) 8 C) 16 D) 108
22	The third proportion of the two numbers 4 and 25 equals .... A) 10 B) 29 C) 100 D) ± 10
23	If the number 6 is the positive proportional mean of two numbers 2 and m , then m = ... A) 8 B) 12 C) 18 D) 36

24	If : $\frac{a}{b} = \frac{b}{c} = \frac{c}{5} = 2$, then a = ... A) 5×2^2 B) 40 C) 10 D) 2×5^3
25	If : 24 , X , 6 and 3 are proportional quantities , then X = ... A) 9 B) 12 C) 18 D) 48
26	If : $\frac{a}{2} = \frac{b}{3}$, then $\frac{b-a}{b+a}$ = ... A) $\frac{1}{5}$ B) $\frac{1}{3}$ C) $\frac{2}{5}$ D) $\frac{3}{5}$

Essay problems:

1	Find the middle proportion between -2 and -8
2	Find the third proportion for 3 and 6
3	If b is the middle proportion between a and c , prove that : $\frac{a}{c} = \frac{b^2}{c^2}$
4	If b is the middle proportion between a and c , prove that : $\frac{a+b}{b+c} = \frac{a}{b}$
5	If b is the middle proportion between a and c , prove that : $\frac{a^3+b^3}{b^3+c^3} = \frac{a^2}{c\,b}$
6	If b is the middle proportion between a and c , prove that : $\frac{a^2}{b^2} + \frac{b^2}{c^2} = \frac{2a}{c}$
7	If a , b , c and d are in continued proportion prove that : $\frac{ab-cd}{b^2-c^2} = \frac{a+c}{b}$
8	If a , b , c and d are in continued proportion prove that : $\frac{a^2+b^2+c^2}{b^2+c^2+d^2} = \frac{ac}{bd}$
9	If a , b , c and d are in continued proportion prove that : $\sqrt[3]{\frac{5a^3-3c^3}{5b^3-3d^3}} = \frac{a+c}{b+d}$
10	If a , b , c and d are in continued proportion prove that : $\frac{a^2+d^2}{c(a+c)} = \frac{b}{d} + \frac{d}{b} - 1$
11	If $\frac{a^2+d^2}{c(a+c)} = \frac{a^2+d^2}{c(a+c)}$, prove that b is the middle proportion between a and c where ac is a positive quantity.

Sheet (8)

Direct Variation & Inverse Variation

First The direct variation

Definition

It is said that y varies directly as x and it is written $y \propto x$ if $y = mx$

i.e. $\frac{y}{x} = m$ (where m is a constant $\neq 0$)

If the variable x took the two values x_1 and x_2 and y took the two values y_1 and y_2 respectively , then : $\frac{y_1}{y_2} = \frac{x_1}{x_2}$

Second The inverse variation

Definition

It is said that y varies inversely as x and it is written $y \propto \frac{1}{x}$ if $y = \frac{m}{x}$

i.e. $xy = m$, where (m is a constant $\neq 0$)

If the variable x took the two values x_1 , x_2 and as a result for that y took the two values y_1 and y_2 respectively , then : $\frac{y_1}{y_2} = \frac{x_2}{x_1}$

Choose the correct answer :

1 The relation which represents the direct variation between X and y is ....

A) $\frac{X}{2} = \frac{y}{3}$ B) $Y = X + 3$ C) $Xy = 3$ D) $\frac{X}{2} = \frac{3}{y}$

2 The relation which represents the direct variation between X and y is ....

A) $Xy = 4$ B) $Y = X + 7$ C) $\frac{X}{2} = \frac{5}{y}$ D) $\frac{X}{3} = \frac{y}{8}$

3 The relation which represents the direct variation between X and y is ....

A) $Xy = 7$ B) $y = X + 2$ C) $\frac{X}{3} = \frac{4}{y}$ D) $y = 2X$

4	The relation which represents the direct variation between X and y is .... A) $y^2 = 2X$ B) $\frac{X}{y} - X = 2$ C) $Xy = 3$ D) $\frac{Y}{X} - 9 = 5$
5	Which of the following relations represents an inverse variation between the two variables X and y ?. A) $y = X + 2$ B) $Y = 4X$ C) $\frac{X}{y} = \frac{5}{7}$ D) $Xy = 11$
6	If : $y \propto X$ and $y = 1$ when $X = 3$, then $y =$ .... When $X = 6$ A) 18 B) 6 C) 2 D) 1
7	If : $y \propto X$ and $y = 5$ when $X = 3$, then : the constant proportional = ... A) 15 B) 5 C) 3 D) $\frac{5}{3}$
8	If : $y \propto X^2$ and $X = 1$ as $y = 2$, then the constant variation is .... A) $\frac{1}{2}$ B) 1 C) 2 D) 32
9	If y varies inversely with X , and $X = \sqrt{3}$ when $y = \frac{2}{\sqrt{3}}$, then : the constant proportion = ... A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) 2 D) 6
10	If y varies inversely as X^2 , k is a constant , then : .... A) $y = k X^2$ B) $Y = k - X^2$ C) $y = \frac{kX}{X^2}$ D) $y = \frac{k}{X^2}$
11	If : $\frac{y}{X} = 5$, then $y \propto$ .... A) X B) $\frac{1}{X}$ C) X^5 D) $\frac{1}{X^5}$
12	If : $X y = 5$, then : $y \propto$ .... A) $\frac{1}{X}$ B) $X - 5$ C) X D) $X + 5$
13	If : $3 X y = 10$, then : $X \propto$ .... A) y^2 B) $\frac{1}{y^2}$ C) y D) $\frac{1}{y}$
14	If : $5 X y = 6$, then .... A) $X \propto y$ B) $y \propto X$ C) $5X \propto 6y$ D) $X \propto \frac{1}{y}$

15 If : $X y^2 = m$ where m is a Constant $\neq 0$, then X varies inversely with ....

A) $\frac{1}{y^2}$ B) $\frac{1}{y}$ C) y D) y^2

16 If : $y = 3X - 6$, then $y \propto$ ....

A) X B) $3X$ C) $X - 2$ D) $3X + 6$

17 If : $(b - 1) = (\frac{1}{a} - \frac{1}{b})$ such that $a \neq b \neq$ zero , then ....

A) $B \propto a + 1$ B) $B \propto a$ C) $B \propto \frac{1}{a}$ D) $B \propto \frac{1}{a^2}$

18 If : $y = \frac{-3}{X}$, then ....

A) $y = X$ B) $y \propto X$ C) $y \propto \frac{1}{X}$ D) $yX = 0$

19 If : $y^2 + 4X^2 = 4Xy$, then ....

A) $y \propto \frac{1}{X}$ B) $y \propto X^2$ C) $y \propto \frac{1}{X^2}$ D) $y \propto X$

20 If : $1 + 4X^2 y^2 = 4Xy$, then ....

A) $y \propto \sqrt{X}$ B) $y \propto \frac{1}{X}$ C) $y \propto X$ D) $y \propto \frac{1}{X^2}$

21 The relation which represents the direct variation between X and y is ....

A) $Xy = \frac{1}{3}$ B) $\frac{y}{7} = \frac{X}{6}$ C) $\frac{X}{2} = \frac{4}{y}$ D) $y = X + 4$

22 Which of the following relations represents an inverse variation between the two variables X and y ?.

A) $Y = \frac{X}{7}$ B) $Xy = 7$ C) $Y = 7X$ D) $\frac{y}{X} = \frac{7}{2}$

23 If : $y \propto X$ and $y = 1$ when $X = 4$, then $y =$ .... When $X = 8$

A) 1 B) 2 C) 4 D) 8

24 If : $y \propto X$ and $X = 2$ when $y = 4$, then the proportion constant = ...

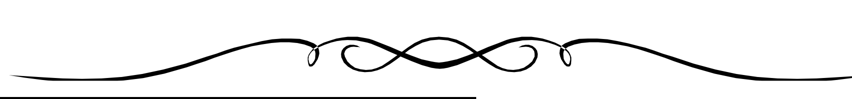
A) 2 B) 4 C) 6 D) 3

Essay problems:

1	If : $y \propto X$ and $y = 14$ when $X = 42$, Find : 1) the relation between X and y 2) the value of y when $X = 60$
2	If : $y \propto$ the multiplicative inverse of the expression $\frac{1}{X^2}$, Find the relation between X and y , if $y = 4$ as $X = 3$, then find the value of y when $X = 9$
3	If : $y \propto (X + 1)$ and $X = 3$ when $y = 2$, then Find the relation between X and y
4	If : $\frac{21X - y}{7X - z} = \frac{y}{z}$, then prove that : $y \propto z$
5	If : $X^4 y^2 - 14 X^2 y + 49 = 0$, then prove that : $y \propto \frac{1}{X^2}$
6	If $y = a - 9$ and $y \propto \frac{1}{X^2}$ and $a = 18$ when $X = \frac{2}{3}$, find the relation between y and X , then deduce the value of y when $X = 1$
7	A car moves with a uniform velocity where the distance varies directly with the time (t). If the car covered a distance of 150 km. in 6 hours, find the distance covered by that car in 10 hours.
8	If the number of hours (n) needed for carrying out a work varies inversely as the number of workers (X) who carry out this work. If the work is carried out by 6 workers within 4 hours , what is the needed time for carrying out the work by 8 workers



Homework



Choose the correct answer :

1	The relation which represents the direct variation between a and b is .... A) ab = 3 B) $\frac{a}{5} = \frac{2}{b}$ C) A + b = 5 D) $\frac{a}{4} = \frac{b}{7}$
2	The relation represents the direct variation between X and y which is .... A) Xy = 5 B) y = X + 3 C) $\frac{X}{3} = \frac{4}{y}$ D) y = 2 X
3	The relation which represents the direct variation between X and y is .... A) $Xy = \frac{1}{3}$ B) $\frac{y}{7} = \frac{X}{6}$ C) $\frac{X}{2} = \frac{4}{y}$ D) y = X + 4
4	Which of the following relations represents an inverse variation between the two variables X and y ?. A) $Y = \frac{X}{7}$ B) Xy = 7 C) Y = 7 X D) $\frac{y}{X} = \frac{7}{2}$
5	If : $y \propto X$ and y = 1 when X = 4 , then y = ... When X = 8 A) 1 B) 2 C) 4 D) 8
6	If : $y \propto X$ and X = 2 when y = 4 , then the proportion constant = ... A) 2 B) 4 C) 6 D) 3
7	If : $y \propto X$ and y = 6 at X = 2 , then y = ... When X = 3 A) 6 B) 9 C) 12 D) 18
8	If y varies inversely with X , and $X = \sqrt{2}$ when $y = \frac{3}{\sqrt{2}}$, then : the relation between X and Y is .... A) Y = 3 X B) X = 3 y C) X y = 3 D) 3 X y = 1
9	If y varies inversely with X , and $X = \sqrt{5}$ when $y = \frac{3}{\sqrt{5}}$, then : the relation between X and Y is .... A) Y = 3 X B) $Xy = \frac{5}{3}$ C) X y = 3 D) $y = \frac{5}{3} X$
10	If : $y \propto \frac{2}{\sqrt{X}}$, then : X varies .... A) Directly as y^2 b) Inversely as y^2 c) Inversely as y d) Inversely as $\sqrt{y}$

11 If : $\frac{a}{b} = \frac{2}{5}$, then : $a \propto$ ....

A) B B) $\frac{1}{b}$ C) b^2 D) $\sqrt{b}$

12 If : Xy = m where $m \neq 0$, then y varies inversely with ....

A) X B) m + X C) $\frac{1}{X}$ D) $\frac{m}{X}$

13 If : 3 X y = 8 , then : ....

A) $X \propto y$ B) $y \propto X$ C) $3X \propto 8y$ D) $X \propto \frac{1}{y}$

14 If : $X y^5$ = Constant , then X varies inversely as ....

A) $\frac{1}{5}$ B) y^5 C) y D) y^2

15 If : $\frac{y+3}{y} = \frac{X+2}{X}$ where $X \neq y \neq$ zero , then : $y \propto$ ....

A) X B) $\frac{1}{X}$ C) X + 2 D) X + 5

16 If : $y - X = \frac{1}{X} - \frac{1}{y}$ where $X \neq y \neq$ zero , then ....

A) $y \propto X + 1$ B) $y \propto X$ C) $y \propto \frac{1}{X}$ D) $y \propto \frac{1}{X^2}$

17 If : y = 5 X , then $y \propto$ ....

A) X B) X + 5 C) $\frac{1}{X}$ D) $\frac{1}{X^2}$

18 If : $y^2 - 4Xy + 4X^2 = 0$, then ....

A) $y \propto X$ B) $y \propto X^2$ C) $y \propto \frac{1}{X}$ D) $y \propto \frac{1}{X^2}$

19 If : $y^2 + 9X^2 = 6Xy$, then ....

A) $y \propto X$ B) $y \propto X^2$ C) $y \propto \frac{1}{X}$ D) $y \propto \frac{1}{X^2}$

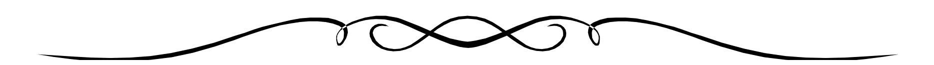
20 If the total cost of a trip is (y) , some of it is constant (a) and the other is directly proportional with the number of participants (X) , then ....

A) y = a X b) $y = \frac{a}{X}$

c) $y = a + \frac{m}{X}$ (m is constant $\neq 0$) d) y=a+m X (m is constant $m \neq 0$)

Essay problems:

1	If : $y \propto \frac{1}{X}$ and y = 3 when X = 2 , Find : 1) the relation between X and y 2) the value of y when X = 1.5
2	If : $y^2 \propto X^3$, Find the relation between X and y where y = 3 as X = 2
3	If : $\frac{a+b}{3} = \frac{2b+c}{6}$, then prove that : $c \propto a$
4	If : $X^2 y^2 - 6Xy + 9 = 0$, then prove that : y varies inversely as X
5	If : X = z + 8 and z varies inversely as y and z = 2 as y = 3 , Find y as X = 3
6	If (h) the height of a right circular cylinder (its volume is constant) varies inversely as the square of radius length (r) and h = 27 cm. when r = 10.5 , Find h when r = 15.75 cm.
7	If the weight of a body on the moon (W) is directly proportional with its weight on the ground (R) If the body weighs 84 kg. , on the ground and its weight on the moon is 14 kg. What will its weight be on the moon if its weight on the ground is 144 kg?
8	If the value of speed v that water passes through a hose nuzzle inversely changes with the square of the hose nuzzle radius length r and v = 5 cm./s. when r = 3 cm., find v when r = 2.5 cm.



Sheet (9)

Mean & Standard Deviasion

Mean : -



Remember that

The mean of a set of values = $\frac{\text{The total of values}}{\text{Number of values}}$

For example :

- If the marks of 5 pupils are : 25 , 23 , 21 , 22 , 24
- Then the mean of marks = $\frac{25 + 23 + 21 + 22 + 24}{5}$ = 23 marks.

Notice that :

$23 \times 5 = 25 + 23 + 21 + 22 + 24$

Finding the mean of data from the frequency table with sets

Example

The following table shows the distribution of the marks of 50 pupils in mathematics :

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	8	12	14	9	7	50

Find the mean of these marks.

Solution

1 Determine the centres of sets according to the rule :

The centre of a set = $\frac{\text{the lower limit + the upper limit}}{2}$

2 Form the vertical table :

Set	Centre of the set « X »	Frequency « f »	$X \times f$
10 –	15	8	120
20 –	25	12	300
30 –	35	14	490
40 –	45	9	405
50 –	55	7	385
Total		50	1700

The mean = $\frac{\text{The sum of } (X \times f)}{\text{The sum of } f} = \frac{1700}{50}$ = 34 marks.

Median : -



Remember that

The median is the middle value in a set of values after arranging it ascendingly or descendingly. such that the number of values which are less than it is equal to the number of values which are greater than it.

- To find the median of a set of values , we do as follows :

We arrange the values ascendingly or descendingly

If the values number is odd , then

The median is the value lying in the middle exactly.

For example :
If the values are
42 , 23 , 17 , 30 and 20
We arrange them ascendingly as follows
17 , 20 , 23 , 30 , 42

The median = 23

If the values number is even , then

The median $= \frac{\text{The sum of the two values lying in the middle}}{2}$

For example :
If the values are
27 , 13 , 23 , 24 , 13 , 21
We arrange them ascendingly as follows
13 , 13 , 21 , 23 , 24 , 27

The median $= \frac{21 + 23}{2} = 22$

Mode : -



Remember that

The mode of a set of values is the most common value in the set, or in other words, it is the value which is repeated more than any other values.

For example :

The mode of the set of the values : 7 , 3 , 4 , 1 , 7 , 9 , 7 , 4 is 7

Dispersion of a set of values

It means the divergence or the differences among its values.

- The dispersion is small if the difference among the values is little while the dispersion is great if the difference among the values is great , the dispersion is zero if all the values are equal.

i.e. The dispersion is a measure that expresses how much the sets are homogeneous.

Remark

If all values (individuals) are equal then the dispersion (σ) is zero

- If the standard deviation equals zero that means the all values are equal , it is the perfect homogeneous case (the vanished dispersion).

Dispersion measurements

1 The range (the simplest measure of dispersion) :

It is the difference between the greatest value and the smallest value in the set.

The range = the greatest value – the smallest value

For example :

- If the values of set A are 60 , 58 , 62 , 61 and 59 $\therefore$ The range = 62 – 58 = 4
- If the values of set B are 72 , 78 , 46 , 65 and 39 $\therefore$ The range = 78 – 39 = 39

So the set B is more divergent than the set A

2 Standard deviation :

First : Calculating the standard deviation of a set of values :

The standard deviation $\sigma = \sqrt{\dfrac{\Sigma (x - \overline{x})^2}{n}}$

Where :

x denotes a value of the values ,

$\overline{x}$ denotes the mean of the values and it is read as x bar ,

n denotes the number of values ,

Σ denotes the summation operation.

Example 1

Calculate the standard deviation of the values : 8 , 9 , 7 , 6 and 5

Solution

(1) We find the mean of the values.

$$\overline{X} = \frac{\Sigma X}{n} = \frac{8+9+7+6+5}{5} = 7$$

(2) We form the following table :

X	$X - \overline{X}$	$(X - \overline{X})^2$
8	$8 - 7 = 1$	1
9	$9 - 7 = 2$	4
7	$7 - 7 = 0$	0
6	$6 - 7 = -1$	1
5	$5 - 7 = -2$	4
Total		**10**

(3) We calculate the standard deviation by substituting in the law :

The standard deviation $(\sigma) = \sqrt{\frac{\Sigma (X - \overline{X})^2}{n}}$

$\therefore$ The standard deviation $(\sigma) = \sqrt{\frac{10}{5}} = \sqrt{2} \simeq 1.4$

Second : Calculating the standard deviation of a frequency distribution :

For any frequency distribution :

The standard deviation $\sigma = \sqrt{\frac{\Sigma (X - \overline{X})^2 k}{\Sigma k}}$

Where :

X represents the value or the centre of the set ,

k represents the frequence of the value or the set ,

Σ k is the sum of frequences and $\overline{X}$ (the mean) $= \frac{\Sigma (X \times k)}{\Sigma k}$

(a) Calculating the standard deviation of a simple frequency distribution :

Example 2

The following table shows the distribution of ages of 20 persons in years :

The age	15	20	22	23	25	30	**Total**
Number of persons	2	3	5	5	1	4	**20**

Find the standard deviation of the ages.

Solution

(1) We find the mean of the ages ($\overline{x}$) by using the following table :

The age (x)	Number of persons (k)	$x \times k$
15	2	30
20	3	60
22	5	110
23	5	115
25	1	25
30	4	120
Total	**20**	**460**

The mean ($\overline{x}$) $= \frac{\Sigma (x \times k)}{\Sigma k} = \frac{460}{20} = 23$ years.

(2) We form the following table :

x	k	$x - \overline{x}$	$(x - \overline{x})^2$	$(x - \overline{x})^2 \times k$
15	2	$15 - 23 = -8$	64	128
20	3	$20 - 23 = -3$	9	27
22	5	$22 - 23 = -1$	1	5
23	5	$23 - 23 = 0$	0	0
25	1	$25 - 23 = 2$	4	4
30	4	$30 - 23 = 7$	49	196
Total	**20**			**360**

(3) We calculate the standard deviation from the law :

Standard deviation (σ) $= \sqrt{\frac{\Sigma (x - \overline{x})^2 \times k}{\Sigma k}} = \sqrt{\frac{360}{20}} = \sqrt{18} \simeq 4.24$ years.

(b) Calculating the standard deviation of a frequency distribution of sets :

Example (3)

The following is the frequency distribution of weekly incentives of 100 workers in a factory :

Incentives in pounds	35 –	45 –	55 –	65 –	75 –	85 –
Number of workers	10	14	20	28	20	8

Find the standard deviation of this distribution.

Solution



The centre of the set = $\frac{\text{lower limit + upper limit}}{2}$

① We find the mean ($\overline{X}$)

by using the following table :

Sets	Centres of sets (X)	Frequence (k)	$X \times$ k
35 –	40	10	400
45 –	50	14	700
55 –	60	20	1200
65 –	70	28	1960
75 –	80	20	1600
85 –	90	8	720
Total		**100**	**6580**

$\therefore$ The mean ($\overline{X}$) = $\frac{\Sigma (X \times k)}{\Sigma k} = \frac{6580}{100} = 65.8$ pounds.

② We form the following table :

X	k	$X - \overline{X}$	$(X - \overline{X})^2$	$(X - \overline{X})^2 \times k$
40	10	40 – 65.8 = – 25.8	665.64	6656.4
50	14	50 – 65.8 = – 15.8	249.64	3494.96
60	20	60 – 65.8 = – 5.8	33.64	672.8
70	28	70 – 65.8 = 4.2	17.64	493.92
80	20	80 – 65.8 = 14.2	201.64	4032.8
90	8	90 – 65.8 = 24.2	585.64	4685.12
Total	**100**			**20036**

③ We calculate the standard deviation by using the law :

$$\text{Standard deviation } (\sigma) = \sqrt{\frac{\Sigma (X - \overline{X})^2 \times k}{\Sigma k}} = \sqrt{\frac{20036}{100}} \simeq 14.15 \text{ pounds.}$$

Choose the correct answer :

1	The most repeated value in a set values represents .... A) Median B) Range C) Mode D) measn
2	The mean for the values : 2 , 5 , 6 and 7 is .... A) 2 B) 3 C) 4 D) 5
3	The arithmetic mean for the values : 3 , 4 , 6 and 7 equals .... A) 5 B) 10 C) 20 D) 40
4	The arithmetic mean for the values : 3 , 5 , 6 , 7 and 9 is .... A) 3 B) 4 C) 12 D) 6
5	The arithmetic mean for the values : 4 , 13 , 18 , 25 , 30 is .... A) 18 B) 26 C) 19 D) 10
6	The mean for the values : 5 , 4 , 2 , 6 , 10 and 3 equals .... A) 5 B) 6 C) 10 D) 15
7	The arithmetic mean for the set of values :7 , 3 , 6 , 9 and 5 is .... A) 3 B) 4 C) 6 D) 12
8	The mean for : 30 , 20 , 50 , 60 is .... A) 25 B) 40 C) 50 D) 55
9	The arithmetic mean for the values : 37 , 10 , 23 , 24 and 16 is .... A) 12 B) 15 C) 22 D) 27
10	If The mean for the values : a , 5 , 8 , 7 , 6 equals 6 then a = ... A) 7 B) 6 C) 5 D) 4
11	If the mean for the numbers : 12 , 17 , 19 , X , 14 is 15 , then X = ... A) 10 B) 12 C) 13 D) 15
12	The difference between the maximum and minimum value for a set of data represents .... A) Mean B) Mode C) Range D) Standard deviation
13	The difference between the greatest and smallest value for a set of data represents .... A) Mean B) Mode C) Range D) Standard deviation

No.	Question
14	The range for the values 2 , 13 , 12 , 16 and 14 is .... A) 2 B) 13 C) 14 D) 16
15	If the range for the values : 2 , 7 , a , 6 is 8 where a > 0 , then a = ... A) 4 B) 9 C) – 1 D) 10
16	The range for the values : 3 , 8 , 5 , 20 , 12 is .... A) 3 B) 20 C) 17 D) 5
17	The range for the values : 5 , 2 , 8 , 12 and 9 is................ A) 2 B) 5 C) 10 D) 15
18	The range for the values : 6 , 8 , 4 , 10 , 2 is .... A) 6 B) 7 C) 4 D) 8
19	The range for the values : 7 , 3 , 6 , 9 , 5 is................ A) 3 B) 9 C) 6 D) 12
20	The range for the values : 7 , 4 , 9 , 5 and 13 is .... A) 6 B) 7 C) 9 D) 5
21	The range for the values : 7 , 8 , 11 , 13 , 6 is .... A) 4 B) 8 C) 7 D) 6
22	The range for the values : 7 , 13 , 16 , 9 and 5 is .... A) 3 B) 4 C) 11 D) 12
23	The range for the values : 7 , 13 , 15 , 8 , 6 is .... A) 6 B) 7 C) 8 D) 9
24	The range for the values : 8 , 3 , 4 , 5 and 9 is .... A) 6 B) 12 C) 3 D) 4
25	The range for the values : 8 , 5 , 10 , 6 and 14 is .... A) 10 B) 9 C) 14 D) 13
26	The range for the values : 11 , 2 , 3 , 6 and 8 is .... A) 9 B) 6 C) 3 D) 5
27	The range of : 15 , 7 , 23 , 35 and 10 is .... A) 7 B) 18 C) 28 D) 35
28	The range for the values : 17 , 13 , 16 , 19 and 15 is .... A) 13 B) 6 C) 32 D) 19

29	The range for the values : 18 , 3 , 8 , 23 and 13 is …………… A) 8 B) 18 C) 20 D) 23
30	The range of the set of the values : 43 , 51 , 55 , 47 , 60 is …………… A) 20 B) 17 C) 9 D) 5
31	The most common measure of dispersion is …………… A) Mean B) Mode C) median D) Standard deviation
32	The most common measure of dispersion and the most accurate is ……… A) Mean B) Range C) median D) Standard deviation
33	From the measures of dispersion is the ……………………………… A) Mean B) Range C) median D) Mode
34	One of the dispersion measurements is ……………………………… A) Mean B) Range C) median D) Mode
35	The simplest and easiest dispersion measure is the ……………………………… A) Mean B) Range C) median D) Mode
36	The common measure of dispersion is ……………………………… A) Mean B) Range C) median D) Mode
37	The standard deviation for the values : 7 , 7 , 7 equals ………………… A) 49 B) 7 C) 3 D) Zero
38	If the set of quantities are equal in values , then ………………………… A) $\sigma = 0$ B) $\overline{X} = 0$ C) $X - \overline{X} > 0$ D) $X - \overline{X} < 0$
39	If all the individuals are equals in values , then ………………………… A) $\overline{X} = 0$ B) $\sigma = 0$ C) $X - \overline{X} > 0$ D) $X - \overline{X} < 0$
40	If the dispersion of a set of values equals zero , then a) the difference between the individuals is great. b) the difference between the individuals is small. c) the arithmetic mean of the set values is zero. d) all the values of the individuals are equal.
41	If : $\sum (X - \overline{X})^2 = 36$ for a set of values whose number is 9 , then $\sigma =$ ……… A) 2 B) 4 C) 18 D) 27
42	Selecting a sample of layers of a statistical society is called ……… sample A) Random B) bunch C) Deliberate D) Class (layer)

43 If 18 is greatest individual of a set of individuals and its range is 6 , then the smallest individual of this set = ...

A) 12 B) 36 C) 8 D) 24

44 The positive square root of the average of squares of deviations of values form its arithmetic mean is called ....

A) Mean B) Range C) median D) Standard deviation

Essay problems:

1. Calculate the standard deviation for the next data : 16 , 32 , 5 , 20 , 27

2. If 5 , 6 , 7 , 8 and 9 are marks of pupil in the math. Exams for 5 months , find the mean and the standard deviation.

3. The following frequency distribution shows the number of children of some families in a new city :

Number of children	0	1	2	3	4	
Number of families	8	16	50	20	6	

Calculate the mean and the standard deviation of the number of children

4. The following are frequency distribution for a number of defective units found in 100 boxes of manufactured units :

Number of defective units	0	1	2	3	4	5
Number of boxes	3	16	17	25	20	19

Find the standard deviation of the defective units.

5. The following frequency distribution shows the ages of 10 students :

Age in years	5	8	9	10	12	Total
Number of families	1	2	3	3	1	10

Calculate the standard deviation to ages in years



GEOMETRY FOR PREPARATORY THREE FIRST TERM

PREPARED BY

Mr. MAHMOUD

Sheet (10)

The main trigonometrical ratios of the acute angle

Sheet (10)
The main trigonometrical ratios of the acute angle

The relation between each of the degrees, the minutes and the seconds

- The degree = 60 minutes.
- The minute = 60 seconds

i.e. The degree = $60 \times 60 = 3600$ seconds.

The main trigonometrical ratios of the acute angle

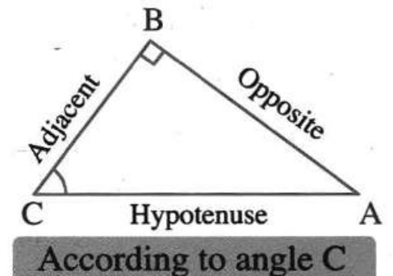
The trigonometrical ratio of the acute angle

It is the ratio between two side lengths of the right-angled triangle that contains this angle.

i.e. If Δ ABC is a right-angled triangle at B , then :

According to angle A

1. $\sin A = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{BC}{AC}$
2. $\cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{AB}{AC}$
3. $\tan A = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{BC}{AB}$



According to angle C

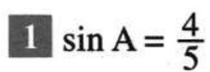
1. $\sin C = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{AB}{AC}$
2. $\cos C = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{BC}{AC}$
3. $\tan C = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{AB}{BC}$

For example

In the opposite figure :

If Δ ABC is a right-angled triangle at B ,

AB = 3 cm. , BC = 4 cm. and AC = 5 cm. , then :



1. $\sin A = \frac{4}{5}$
2. $\cos A = \frac{3}{5}$
3. $\tan A = \frac{4}{3}$

1. $\sin C = \frac{3}{5}$
2. $\cos C = \frac{4}{5}$
3. $\tan C = \frac{3}{4}$

We can deduce that :

The sine of any angle equals the cosine of its complementary and vice versa

i.e. If ∠ A and ∠ B are acute angles , and sin A = cos B

then : m (∠ A) + m (∠ B) = 90°

Generally

The tangent of the angle = $\frac{\text{The sine of the angle}}{\text{The cosine of the angle}}$

Choose the correct answer :

1 cos 46° 32̂ = ... rounding the sum to the nearest four decimal numbers.

(a) 0.6879 (b) 0.5879 (c) 0.7861 (d) 0.8732

2 For any acute angle A , tan A = ... *(El-Ismailia 2012)*

(a) $\frac{\cos A}{\sin A}$ (b) sin A cos A (c) $\frac{\sin A}{\cos A}$ (d) sin A + cos A

3 If Δ ABC is a right-angled at B , then : sin A + cos A ....

(a) equals 0 (b) equals 1 (c) is less than 1 (d) is more than 1

4 If : sin A = cos A , then measure angle A = ...

(a) 30° (b) 45° (c) 60° (d) 90°

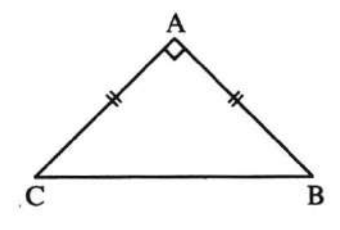
5 tan H × cos H = ...

(a) cos H (b) $\frac{1}{\cos H}$ (c) $\frac{1}{\sin H}$ (d) sin H

6 **In the opposite figure :**

ABC is a right-angled triangle at A ,

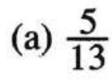
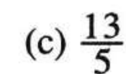
AB = AC , tan C = ...



(a) 1 (b) $\frac{1}{2}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{3}}$

7 In the opposite figure :
ABC is a triangle in which :
m (∠ A) = 90°, AB = 5 cm.
and BC = 13 cm.
, then : tan B = ………
(Qena 2011)

(a)  $\frac{5}{13}$ (b) 2.4 (c)  $\frac{13}{5}$ (d) $\frac{25}{13}$

8 For any acute angle A , tan A = ……… *(El-Ismailia 2012)*

(a) $\frac{\cos A}{\sin A}$ (b) sin A cos A (c) $\frac{\sin A}{\cos A}$ (d) sin A + cos A

9 tan 75° = ……… *(El-Ismailia 2011)*

(a) $\frac{\cos 75°}{\sin 75°}$ (b) $\frac{\sin 75°}{\cos 75°}$ (c) 3 tan 25° (d) 3 sin 25° cos 25°

10 In Δ ABC, if m (∠ A) = 85° and sin B = cos B, then m (∠ C) = ………° *(El-Monofia 2015)*

(a) 30° (b) 45° (c) 50° (d) 60°

11 In Δ ABC : if m (∠ B) = 90° , then sin A + cos C = ………

(a) 2 sin A (b) 2 sin C (c) 2 sin B (d) 2 cos A

(New valley 2015 – Port Said 2014 – El-Fayoum 2013)

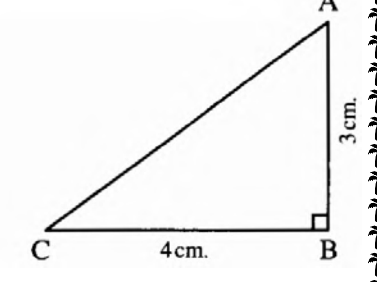
Essay problems:

1 If : tan (x + 10°) = $\sqrt{3}$ where x is an acute angle **find :** x

2 XYZ is a right-angled triangle at Y , where XY = 5 cm. , XZ = 13 cm.
Find the value of :
(1) tan X × tan Z (2) $\sin^2 Z + \sin^2 X$

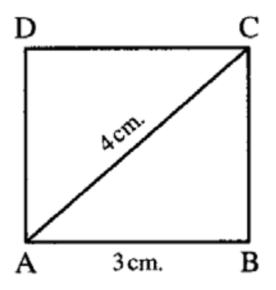
3 ABC is a right-angled triangle at B , if AB : AC = 3 : 5 *(Aswan 2013)*
Find : the main trigonometrical of ∠ A « $\frac{4}{5}$, $\frac{3}{5}$, $\frac{4}{3}$ »

4 **In the opposite figure:**
The triangle ABC is a right-angled at B
AB = 3 cm. and BC = 4 cm.
Find : (1) tan A (2) cos A



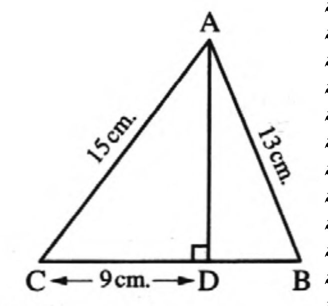
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In the opposite figure :
ABCD is a rectangle where :
AB = 3 cm. , AC = 4 cm.
Find: m (∠ BAC)



6

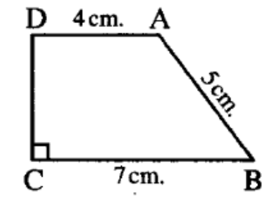
In the opposite figure :
$\overline{AD} \perp \overline{BC}$,
AB = 13 cm. ,
AC = 15 cm. ,
CD = 9 cm.

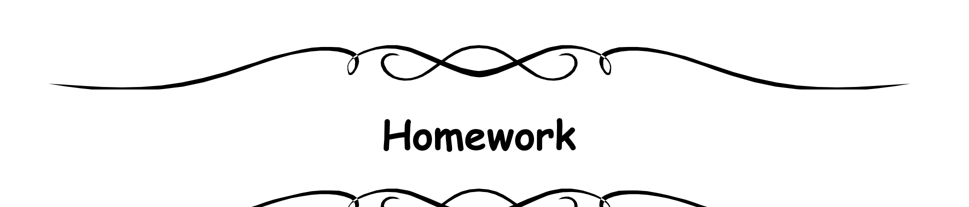


Find in the simplest form the value of : $\frac{\tan(\angle CAD) - \tan(\angle BAD)}{\tan(\angle CAD) + \tan(\angle BAD)}$

7

In the opposite figure : ABCD is right-angled trapezium at C , $\overline{AD} \parallel \overline{BC}$,
AB = 5 cm. , BC = 7 cm. , AD = 4 cm.
Find : (1) sin B , then deduce m (∠ B)
(2) Surface area of trapezium ABCD
(Hint draw $\overrightarrow{AH} \perp \overline{BC}$ to cut it at H)





Homework

Choose the correct answer :

1 For any acute angle A , tan A = ... *(El-Ismailia 2012)*

(a) $\frac{\cos A}{\sin A}$ (b) sin A cos A (c) $\frac{\sin A}{\cos A}$ (d) sin A + cos A

2 For any acute angle A , tan A = ... *(El-Ismailia 2012)*

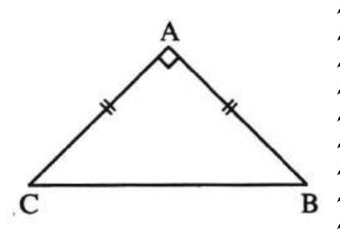
(a) $\frac{\cos A}{\sin A}$ (b) sin A cos A (c) $\frac{\sin A}{\cos A}$ (d) sin A + cos A

3

In the opposite figure :

ABC is a right-angled triangle at A ,

AB = AC , tan C = ...



(a) 1 (b) $\frac{1}{2}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{3}}$

4

In Δ ABC , if m (∠ A) = 85° and sin B = cos B , then m (∠ C) = ...° *(El-Monofia 2015)*

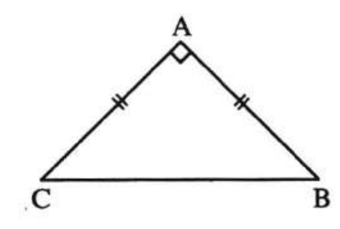
(a) 30° (b) 45° (c) 50° (d) 60°

5

In the opposite figure :

ABC is a right-angled triangle at A ,

AB = AC , tan C = ...



(a) 1 (b) $\frac{1}{2}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{3}}$

6

$\cos 46° 32'$ = ... rounding the sum to the nearest four decimal numbers.

(a) 0.6879 (b) 0.5879 (c) 0.7861 (d) 0.8732

7

tan H × cos H = ...

(a) cos H (b) $\frac{1}{\cos H}$ (c) $\frac{1}{\sin H}$ (d) sin H

8

tan 75° = ... *(El-Ismailia 2011)*

(a) $\frac{\cos 75°}{\sin 75°}$ (b) $\frac{\sin 75°}{\cos 75°}$ (c) 3 tan 25° (d) 3 sin 25° cos 25°

9

If Δ ABC is a right-angled at B , then : sin A + cos A ....

(a) equals 0 (b) equals 1 (c) is less than 1 (d) is more than 1

10

tan H × cos H = ...

(a) cos H (b) $\frac{1}{\cos H}$ (c) $\frac{1}{\sin H}$ (d) sin H

11

If : sin A = cos A , then measure angle A = ...

(a) 30° (b) 45° (c) 60° (d) 90°

12 For any acute angle A , tan A = ………… *(El-Ismailia 2012)*

(a) $\frac{\cos A}{\sin A}$ (b) sin A cos A (c) $\frac{\sin A}{\cos A}$ (d) sin A + cos A

13 If : sin A = cos A , then measure angle A = ……………

(a) 30° (b) 45° (c) 60° (d) 90°

14 cos 46° 32ʹ = …………… rounding the sum to the nearest four decimal numbers.

(a) 0.6879 (b) 0.5879 (c) 0.7861 (d) 0.8732

15 If Δ ABC is a right-angled at B , then : sin A + cos A ……………

(a) equals 0 (b) equals 1 (c) is less than 1 (d) is more than 1

16 **In the opposite figure :**

ABC is a triangle in which :

m (∠ A) = 90° , AB = 5 cm.

and BC = 13 cm.

, then : tan B = ………… *(Qena 2011)*

(a) $\frac{5}{13}$ (b) 2.4 (c) $\frac{13}{5}$ (d) $\frac{25}{13}$

17 In Δ ABC : if m (∠ B) = 90° , then sin A + cos C = …………

(a) 2 sin A (b) 2 sin C (c) 2 sin B (d) 2 cos A

(New valley 2015 – Port Said 2014 – El-Fayoum 2013)

Essay problems:

1 If 3 x is an acute angle , tan 3 $x = \sqrt{3}$, find the measure of angle x .

2 XYZ is a right-angled triangle at Z , XZ = 2 cm. , YZ = 4 cm.

Calculate the numerical value of the quantity :

cos X cos Y + sin X sin Y

3 ABC is a right-angled triangle at C , AB = 5 cm. , BC = 3 cm.

Find : (1) the length $\overline{AC}$ (2) sin A , sin B , tan A tan B

4 ABC is a right-angled triangle at B , where AB = 5 cm. , AC = 13 cm.

(1) **Find :** m (∠ C)

(2) **Prove that :** sin A cos C + cos A sin C = 1

5

If : XYZ is right-angled triangle at Y , where XY = 8 cm. , YZ = 15 cm.

(1) **Find :** $\tan Z$

(2) **Prove that :** $\sin^2 X + \cos^2 X = 1$

6

ABC is right-angled triangle at C , AB = 13 cm. , BC = 12 cm.

Prove that : $\sin A \cos B + \cos A \sin B = 1$

7

ABC is a right-angled triangle at B , if AB : AC = 3 : 5 *(Aswan 2013)*

Find : the main trigonometrical of $\angle A$ « $\frac{4}{5}$, $\frac{3}{5}$, $\frac{4}{3}$ »

8

ABC is an isosceles triangle in which : AB = AC and $\sin \frac{A}{2} = \frac{4}{5}$ *(Red Sea 2013)*

Find cos B without using the calculator. « $\frac{4}{5}$ »

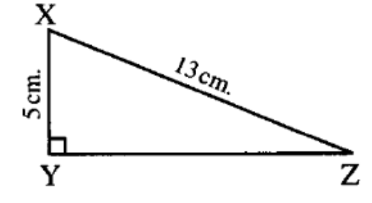
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In the opposite figure :

XYZ is a triangle , m (∠ Y) = 90°

XY = 5 cm. , XZ = 13 cm.

Find: $\sin X \cos Z + \cos X \sin Z$



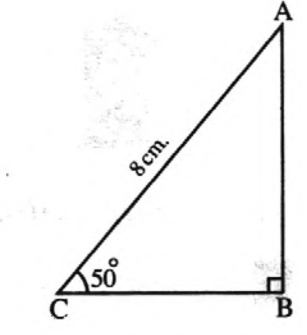
10

In the opposite figure :

m (∠ C) = 50°

, AC = 8 cm.

Find to the nearest one decimal place the length of $\overline{AB}$ and the length of $\overline{BC}$ to the nearest cm.



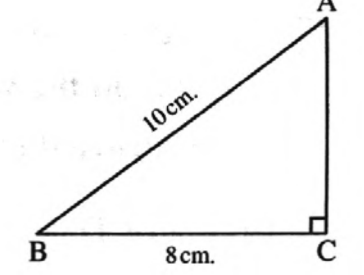
11

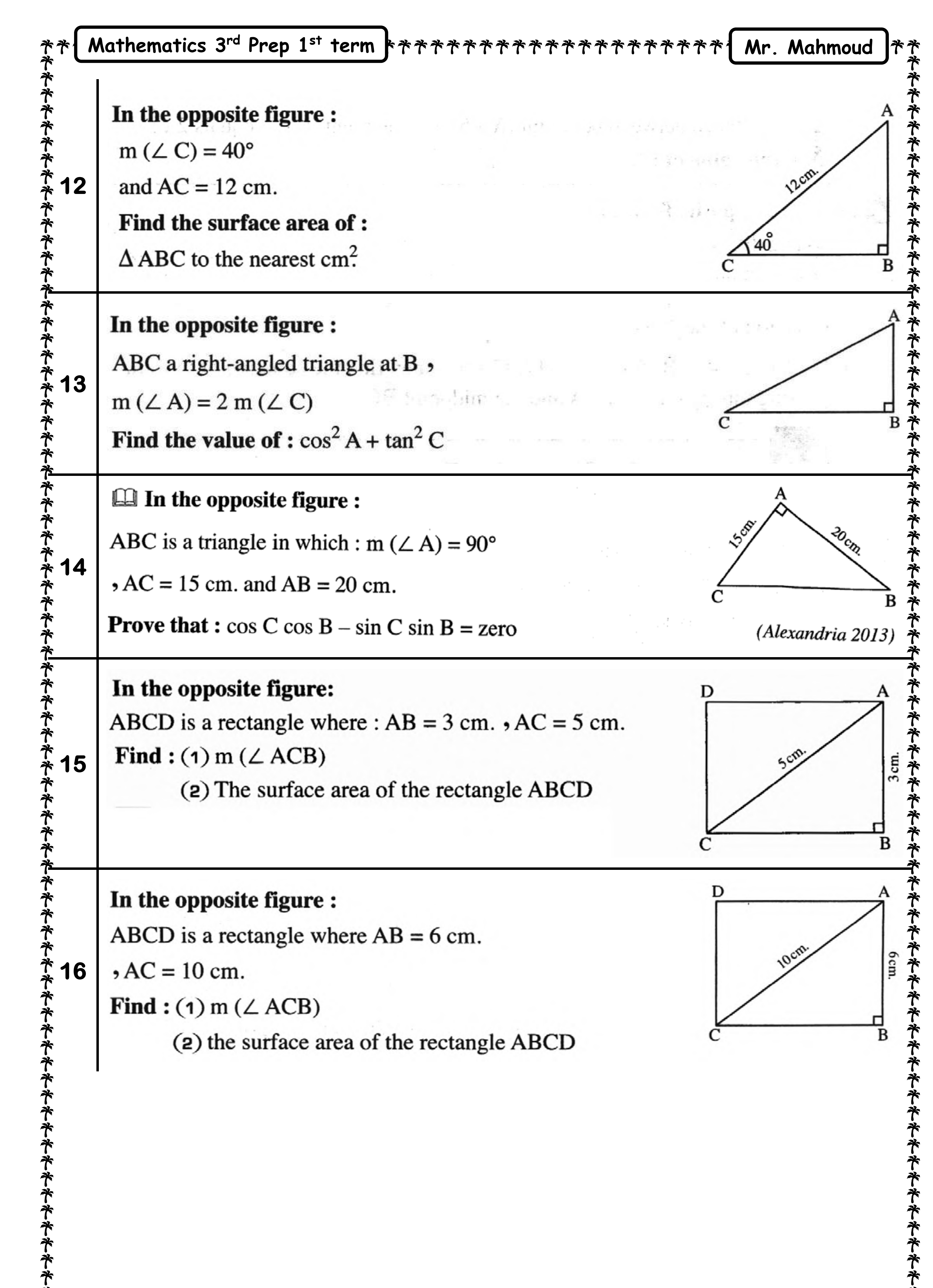
In the opposite figure :

Δ ABC is a right-angled triangle at C ,

AB = 10 cm. and BC = 8 cm.

Calculate the value of : $\sin A \cos B + \cos A \sin B$





12

In the opposite figure :

$m(\angle C) = 40°$

and AC = 12 cm.

Find the surface area of :

ΔABC to the nearest cm^2.

13

In the opposite figure :

ABC a right-angled triangle at B ,

$m(\angle A) = 2\, m(\angle C)$

Find the value of : $\cos^2 A + \tan^2 C$

14

In the opposite figure :

ABC is a triangle in which : $m(\angle A) = 90°$

, AC = 15 cm. and AB = 20 cm.

Prove that : $\cos C \cos B - \sin C \sin B = \text{zero}$

(Alexandria 2013)

15

In the opposite figure:

ABCD is a rectangle where : AB = 3 cm. , AC = 5 cm.

Find : (1) $m(\angle ACB)$

(2) The surface area of the rectangle ABCD

16

In the opposite figure :

ABCD is a rectangle where AB = 6 cm.

, AC = 10 cm.

Find : (1) $m(\angle ACB)$

(2) the surface area of the rectangle ABCD

17

In the opposite figure :

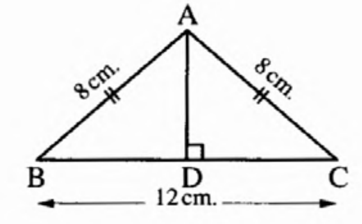
ABC is an isosceles triangle in which :

AB = AC = 8 cm. , BC = 12 cm.

and $\overline{AD} \perp \overline{CB}$

Find : (1) m (∠B)

(2) The area of the surface of the triangle ABC



18

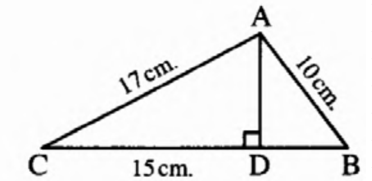
In the opposite figure :

$\overline{AD} \perp \overline{BC}$, AC = 17 cm. ,

DC = 15 cm. , AB = 10 cm.

Find the value of :

3 tan (∠ C) + sin (∠ B)



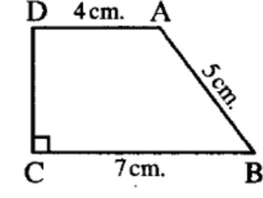
19

In the opposite figure : ABCD is right-angled trapezium at C , $\overline{AD} // \overline{BC}$,
AB = 5 cm. , BC = 7 cm. , AD = 4 cm.

Find : (1) sin B , then deduce m (∠ B)

(2) Surface area of trapezium ABCD

(Hint draw $\overrightarrow{AH} \perp \overline{BC}$ to cut it at H)



Sheet (11)

The main Trigonometrical Ratios of Some Angles

Sheet (11) The main Trigonometrical Ratios of Some Angles

The following table summarizes the trigonometrical ratios of the angles measuring 30° , 60° and 45° :

The measure of the angle / The trigonometrical ratio	30°	60°	45°
sin	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
tan	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	1

Using the calculator

First: Finding the main trigonometrical ratios of a given angle :

In the calculator , there are three keys : [sin] , [cos] , [tan]

1 The key [sin] means sine.

2 The key [cos] means cosine.

3 The key [tan] means tangent.

Example 4 **By using the calculator , find the value of each of the following approximated to the nearest 4 decimals :**

1 $\sin 36°$ **2** $\cos 72° 35'$ **3** $\tan 50° 46' 25''$

Solution Use the keys of the calculator as the following sequence from left :

1 [sin] [3] [6] [=] $\therefore \sin 36° \simeq 0.5878$

2 [cos] [7] [2] [°’’’] [3] [5] [°’’’] [=] $\therefore \cos 72° 35' \simeq 0.2993$

3 [tan] [5] [0] [°’’’] [4] [6] [°’’’] [2] [5] [°’’’] [=]

$\therefore \tan 50° 46' 25'' \simeq 1.2250$

Second: Finding the measure of the angle if one of its trigonometrical ratios is given :

[SHIFT] [sin] [.] [6] [2] [1] [8] [=] [°’’’] Then $A \simeq 38° 26' 52''$

Example 5 Find A in each of the following , where A is the measure of an acute angle :

1 $\sin A = 0.8$ **2** $\cos A = 0.7152$ **3** $\tan A = 1.5156$

Solution Use the keys of the calculator as the following sequence from left :

1 SHIFT sin . 8 = ° ′ ″ $\therefore A \simeq 53° \, 7' \, 48''$

2 SHIFT cos . 7 1 5 2 = ° ′ ″ $\therefore A \simeq 44° \, 20' \, 25''$

3 SHIFT tan 1 . 5 1 5 6 = ° ′ ″ $\therefore A \simeq 56° \, 34' \, 59''$

Choose the correct answer :

1 $\tan 75° = \dots\dots$ *(El-Ismailia 2011)*

(a) $\frac{\cos 75°}{\sin 75°}$ (b) $\frac{\sin 75°}{\cos 75°}$ (c) $3 \tan 25°$ (d) $3 \sin 25° \cos 25°$

2 $\sin 60° + \cos 30° - \tan 60° = \dots\dots$

(a) zero (b) $\sqrt{3}$ (c) $2\sqrt{3}$ (d) 1

3 $\sin^2 45° + \cos^2 45° = \dots\dots$

(a) $2\sqrt{2}$ (b) 0 (c) -1 (d) 1

4 If x is the measure of an acute angle and $\sin x = \frac{1}{2}$, then : $\sin 2x = \dots\dots$ *(El-Ismailia 2012)*

(a) 1 (b) $\frac{1}{4}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

5 If $\sin 30° = \cos \theta$, where θ is an acute angle , then $m(\angle \theta) = \dots\dots$ °

(a) 60 (b) 45 (c) 10 (d) 30

6 $\cos(3x + 6°) = \sin 30°$ such that $(3x + 6°)$ is an acute angle , then the value of $x = \dots\dots$

(a) 60° (b) 54° (c) 36° (d) 18°

7 If : $2 \sin x = 1$ where $0° < x < 90°$, then $m(\angle x) = \dots\dots$°

(a) 60 (b) 45 (c) 30 (d) 50

8 If : $\sin \frac{x}{2} = \frac{1}{2}$ such that $\left(\frac{x}{2}\right)$ is the measure of an acute angle , then $\tan x = \dots\dots$

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $\frac{\sqrt{3}}{2}$

9 If : $\sqrt{2}\cos 3X = 1$, where X is measure of an acute angle , then X = ...°

(a) 15 (b) 30 (c) 45 (d) 60

10 If : $\tan X = 2\sin 60°$, where $0° < X < 90°$, then X = ... °

(a) 60 (b) 45 (c) 30 (d) 15

11 If : $\cos X = \frac{1}{2}$ where X is an acute angle , then : m ($\angle X$) = ... *(Cairo 2013)*

(a) 90° (b) 60° (c) 45° (d) 30°

12 If $\sin X = \frac{1}{2}$ where X is an acute angle , then m ($\angle X$) = ... *(Cairo 2015)*

(a) 90° (b) 60° (c) 45° (d) 30°

13 If $2\sin X = \tan 60°$ where X is an acute angle , then m ($\angle X$) = ... *(Souhag 2011)*

(a) 30° (b) 45° (c) 60° (d) 40°

14 If $\tan 3X = \sqrt{3}$ where $3X$ is an acute angle , then m ($\angle X$) = ... *(Ismailia 2015)*

(a) 20° (b) 30° (c) 45° (d) 60°

15 If $\sin 2X = \frac{\sqrt{3}}{2}$, then X = ... (where $2X$ is an acute angle) *(Giza 2011)*

(a) 20° (b) 30° (c) 45° (d) 60°

16 If $\cos(X + 10°) = \frac{1}{2}$ where $(X + 10°)$ is an acute angle , then X = ... *(El-Fayoum 2011)*

(a) 30° (b) 40° (c) 50° (d) 70°

17 If $\sin(X + 5°) = \frac{1}{2}$ where $(X + 5°)$ is the measure of an acute angle , then $\tan(X + 20°)$ = ... *(El-Dakahlia 2011)*

(a) $\frac{\sqrt{2}}{2}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

18 If X and y are complementary angles where $X : y = 1 : 2$, then $\sin X + \cos y$ = ...

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

(El-Beheira 2015)

Essay problems:

1. **Find the numerical value of :** $2 \sin 45° \cos 45° + 4 \sin 30° \cos 60°$

2. Without using calculator find the value of : $\tan^2 45° - 4 \sin^2 30°$

3. **Prove that :** $2 \sin 45° \cos 45° = \sin 90°$

4. **Find the value of X in each of the following :**

   $2 \sin X = \sin 30° \cos 60° + \cos 30° \sin 60°$ where X is an acute angle. *(Cairo 2011)* « 30° »

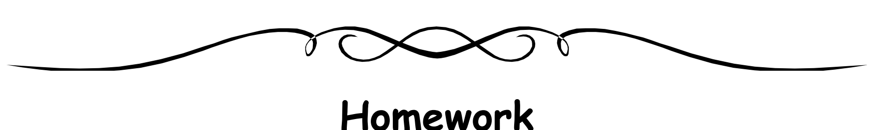
5. **Find the value of :** $X°$, where $0° < X < 90°$

   If $\sin X \sin 45° \cos 45° \tan 60° = \tan^2 45° - \cos^2 60°$

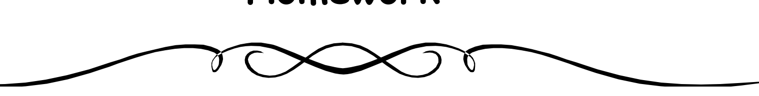
6. Find the value of : A (Where A is an acute angle) which satisfies

   $2 \sin A = \tan^2 60° - 2 \tan 45°$

7. Without using the calculator . **Prove that :** $\cos 60° + 2 \sin^2 45° = \sin 30° + 3 \tan^2 30°$



Homework



Choose the correct answer :

1. $\tan 45° = \dots\dots$

   (a) $\sqrt{3}$ (b) $\frac{1}{2}$ (c) 1 (d) $\frac{1}{\sqrt{3}}$

2. $\tan 75° = \dots\dots$ *(El-Ismailia 2011)*

   (a) $\frac{\cos 75°}{\sin 75°}$ (b) $\frac{\sin 75°}{\cos 75°}$ (c) $3 \tan 25°$ (d) $3 \sin 25° \cos 25°$

3. The value of : $\sin^2 30° + \cos^2 30° = \dots\dots$

   (a) zero (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1

No.	Question
4	$\sin^2 60° + \cos^2 30° - \tan 45° =$ .... (a) zero (b) $\frac{1}{2}$ (c) 2 (d) 3
5	$\sin^2 60° - \cos^2 60° =$ .... (a) zero (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1
6	$\sin 60° =$ .... (a) 1 (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{3}}$
7	$\sin 45° \cos 45° =$ .... (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) 1
8	$\tan 45° \sin 30° =$ .... (a) $\frac{1}{2}$ (b) 1 (c) $\frac{2}{3}$ (d) $\frac{1}{4}$
9	$2 \sin 30° \cos 30° =$ .... (a) $\sin 60°$ (b) $\cos 60°$ (c) $\tan 60°$ (d) $2 \sin 60°$
10	$4 \cos 30° \sin 60° =$ .... (a) 6 (b) $2\sqrt{3}$ (c) 3 (d) 12
11	$\sqrt{2} \sin 45° \cos 30° =$ ....° (a) $\sin 60°$ (b) $\cos 60°$ (c) $\tan 60°$ (d) $2 \sin 60°$
12	$\sin 45° + \cos 45° =$ .... (a) 2 (b) $\frac{1}{2}$ (c) $\sqrt{2}$ (d) $\sqrt{3}$
13	$2 \tan 45° - \frac{1}{\sin 30°} =$ .... (a) 0 (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1
14	$\frac{\sin 50°}{\cos 40°} = \tan$ ....° (a) 50 (b) 45 (c) 40 (d) 30

15 If : $\sin X = \frac{\sqrt{3}}{2}$, where X is an acute angle , then $X =$ ....

(a) 30° (b) 45° (c) 60° (d) 90°

16 If : $\cos X = \sin 45°$, where X is the measure of an acute angle , then $X =$ .... °

(a) 15 (b) 30 (c) 45 (d) 60

17 If $2 \sin X = \tan 60°$ where X is an acute angle , then : $m(\angle X) =$ .... *(Souhag 2011)*

(a) 30° (b) 45° (c) 60° (d) 40°

18 If $\sin 2X = \frac{\sqrt{3}}{2}$, then : $X =$ .... (where $2X$ is an acute angle) *(Giza 2011)*

(a) 20° (b) 30° (c) 45° (d) 60°

19 If : $\sin (X + 5°) = \frac{1}{2}$, where $(X + 5°)$ is an acute angle , then : $X =$ ....°

(a) 5 (b) 10 (c) 25 (d) 30

20 If $\sin (X + 5°) = \frac{1}{2}$ where $(X + 5°)$ is the measure of an acute angle , then : $\tan (X + 20°) =$ .... *(El-Dakahlia 2011)*

(a) $\frac{\sqrt{2}}{2}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

21 If : $\sin (y + 7°) = 0.5$, then $y =$ .... °

(a) 23 (b) 30 (c) 53 (d) 7

22 If : $\sin X = 2 \cos 60° \sin 30°$ where X is acute angle , then $m(\angle X) =$ .... °

(a) 30 (b) 60 (c) 45 (d) 75

23 If : $\cos 2X = \frac{1}{2}$ where X is an acute angle , then $m(\angle X) =$ ....

(a) 15° (b) 30° (c) 45° (d) 60°

24 If $\cos 3X = \frac{1}{2}$ where $(3X)$ is an acute angle , then : $X =$ ....

(a) 5 (b) 10 (c) 15 (d) 20

25 If : $\cos (X + 5) = \frac{1}{2}$ where $(X + 5)$ is an acute angle , then $X =$ .... °

(a) 10 (b) 55 (c) 25 (d) 30

26 If $\cos(X + 10°) = \frac{1}{2}$ where $(X + 10°)$ is an acute angle , then : $X =$ .... *(El-Fayoum 2011)*

(a) 30° (b) 40° (c) 50° (d) 70°

27 If : $\cos(X + 20°) = \frac{1}{2}$ where X is an acute angle, then : $m(\angle X)$ equals .... °

(a) 10 (b) 25 (c) 40 (d) 60

28 If : $\cos\frac{X}{2} = \frac{\sqrt{3}}{2}$, then $X =$ ....°

(a) 30 (b) 45 (c) 60 (d) 90

29 $2\cos 30° \tan 60° =$ ....

(a) $2\sqrt{3}$ (b) 6 (c) 3 (d) 9

30 $4\cos 30 \tan 60° =$ ....

(a) 12 (b) 3 (c) 4 (d) 6

31 If : $\tan 3X = \sqrt{3}$ where $3X$ is an acute angle , then : $m(\angle X) =$ .... *(Suez 2013)*

(a) 10° (b) 20° (c) 40° (d) 60°

32 If : $\tan\frac{1}{2}X = \frac{1}{\sqrt{3}}$ where X is an acute angle , then : $m(\angle X) =$ ....

(a) 30° (b) 60° (c) 90° (d) 45°

33 If : $\tan(X + 20°) = 1$, then : $m(\angle X) =$ ....

(a) 60° (b) 30° (c) 45° (d) 25°

34 If : $\tan(X + 10°) = \sqrt{3}$ where X is an acute angle , then $m(\angle X) =$ .... °

(a) 50 (b) 35 (c) 20 (d) zero

Essay problems:

1 **Without using the calculator find the value of :**

$(\cos 30° - \cos 60°)(\sin 60° + \sin 30°)$

2 **Without using the calculator find the value of :**

$\sin 30° \cos 60° - \sin 60° \tan 60° + \cos^2 30°$

3	**Find the value of :** $\cos^2 45° + \tan^2 60° - \sin 30°$
4	**Without using calculator , find the numerical value of the expression :** $\cos 60° \sin 30° - \sin 60° \cos 30°$
5	**Prove that :** $\sin 45° \cos 45° + \sin 30° \cos 60° = \cos^2 30°$
6	**Without using a calculator , prove that :** $\sin 30° \cos 60° + \cos^2 30° + 2 \tan 45° = 6 \sin^2 45°$
7	**Without using calculator , prove that :** $\sin^2 60° = 5 \sin^2 30° - \sin^2 45°$
8	**Without using calculator , prove that :** $\frac{1}{2} \sin^2 45° \tan^2 60° - 3 \sin^2 60° \tan^2 30° = 0$
9	**Prove that :** $\cos 60° = \cos^2 30° - \sin^2 30°$
10	**Without using the calculator, prove each of the following :** $\cos^2 60° = 5 \sin^2 30° - \tan^2 45°$ *(Kafr El-Sheikh 2011)*
11	**Prove that :** $\cos 60° = 2 \cos^2 30° - 1$
12	**Without using the calculator, prove each of the following :** $\tan 60° = \frac{2 \tan 30°}{1 - \tan^2 30°}$ *(Beni Suef 2011)*
13	**Prove that :** $\tan 60° = 2 \tan 30° \div (1 - \tan^2 30°)$
14	**Without using the calculator find the value of x where :** $x = \cos^2 30° + \sin^2 30° + \tan^2 60°$
15	**Find the value of x in each of the following :** $3 \sin x = \sin 45° \cos 45° + \sin 30° \cos 60° + \cos^2 30°$ where x is an acute angle. *(Port Said 2011)* « 30° »

16 Find the value of X in each of the following :

sin X = sin 60° cos 30° – cos 60° sin 30° where X is an acute angle.

(El-Kalyoubia 2011) « 30° »

17 **Find the value of E where :** $2 \cos E = 4 \sin^2 60° - 2 \tan 45°$ where : E is an acute angle.

18 If : $\cos X = \tan 45° \cos^2 30° - \sin^2 30°$ **Find the value of :** X where $0 < X < 90°$

19 **Find the value of X that satisfies that :** $\cos X = \dfrac{\sin 60° \sin 30°}{\tan 45° \sin^2 45°}$

where X is the measure of an acute angle.

20 **Prove that :** $\sin 60° = 2 \sin 30° \cos 30°$

21 Find the value of X in each of the following :

$\tan X = 4 \sin 30° \cos 60°$ where X is an acute angle. *(Assiut 2011)* « 45° »

22 If : $\cos N = \dfrac{(\sin 60°)^2 - \sin 30°}{3 (\cos 45°)^2 + 1}$, where $0 < N < 90°$

Find : m (∠ N) in degrees.

23 If : $2\sin \theta = 4 \cos 60° - \tan 45°$

Find the value of θ where θ is measure of an acute angle.

Sheet (12)

Distance Between Two Points

i.e. The distance between the two points M and N equals $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

and we know that :

$(x_2 - x_1)^2 = (x_1 - x_2)^2$, and similarly : $(y_2 - y_1)^2 = (y_1 - y_2)^2$, therefore :

The distance between the two points M and N equals also $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Generally : The distance between two points =

$\sqrt{\text{square of the difference between } x\text{-coordinates} + \text{square of the difference between } y\text{-coordinates}}$

For example :

- The distance between the two points M (3 , 6) and N (– 1 , 4) is :

$$MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-1-3)^2 + (4-6)^2} = \sqrt{(-4)^2 + (-2)^2}$$

$$= \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5} \text{ length unit.}$$

Remark 1

To prove that three given points are collinear (***i.e.*** they lie on one straight line) we can find the distance between each two of these points , then prove that the greatest distance equals the sum of the two other distances.

Remark 2

- To prove that the points : A , B and C are vertices of a triangle , we can find AB , BC and AC , then prove that the sum of the smaller two lengths is greater than the third length.
- To determine the type of the triangle ABC according to its angle measures (where $\overline{AC}$ is the longest side of the triangle ABC)

  We compare between $(AC)^2$ and $(AB)^2 + (BC)^2$ as the following :

  1 If $(AC)^2 > (AB)^2 + (BC)^2$, then the triangle is obtuse-angled at B

  2 If $(AC)^2 = (AB)^2 + (BC)^2$, then the triangle is right-angled at B

  3 If $(AC)^2 < (AB)^2 + (BC)^2$, then the triangle is acute-angled.

Remark 3

If ABCD is a quadrilateral :

1 To prove that ABCD is a parallelogram , we prove that : AB = CD , BC = AD

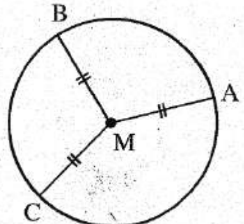
2 To prove that ABCD is a rhombus , we prove that : AB = BC = CD = DA

3 To prove that ABCD is a rectangle , we prove that : AB = CD , BC = AD , AC = BD

4 To prove that ABCD is a square , we prove that : AB = BC = CD = DA , AC = BD

Remark 4

- To prove that : Three points as A , B and C lie on a same circle of centre M
  we prove that : MA = MB = MC
- If A $\in$ the circle M , then the radius length of this circle (r) = MA
- Remember that :
  - Circumference of the circle = $2\pi r$
  - Area of the circle = πr^2



Lesson [2] : Two Coordinates Of Midpoint Of A Line Segment

First point : A (X_1 , y_1) , **Second point : A (X_2 , y_2)**

Midpoint point: M (m_x , m_y) then

$$M(m_x, m_y) = \left(\frac{X_1 + X_2}{2}, \frac{y_1 + y_2}{2}\right),$$

$X_1 = m_x \times 2 - X_2$ $y_1 = m_y \times 2 - y_2$

For Example : -

- If A (1 , 5) , B (3 , 1) and M is the midpoint of $\overline{AB}$, then :

$$M = \left(\frac{1+3}{2}, \frac{5+1}{2}\right) = (2, 3)$$

- If X (3 , – 2) , Y (– 1 , – 4) and M is the midpoint of $\overline{XY}$, then :

$$M = \left(\frac{3+(-1)}{2}, \frac{-2+(-4)}{2}\right) = (1, -3)$$

Remark : -

If $\overline{AB}$ is a diameter in a circle of centre M , then M is the midpoint of $\overline{AB}$

Choose the correct answer :

1	The length of the line segment drawn from the point (0 , 0) to the point (– 4 , 3) equals .... units of length. (a) 3 (b) 4 (c) $\sqrt{7}$ (d) 5
2	The distance between the two points (5 , 0) and (0 , – 12) = ... length units. (a) 7 (b) 13 (c) 17 (d) 10
3	If : C (– 6 , 0) , D (0 , 8) , then CD = ... length unit. (a) 14 (b) 10 (c) 5 (d) 2
4	The distance between the point (2 , –3) and the X-axis equals .... units length. (a) 2 (b) 3 (c) –3 (d) –2
5	The distance between the point (2 , – 5) and the X-axis = ... length unit. (a) 5 (b) 2 (c) – 5 (d) – 3
6	The distance between the point (3 , – 5) and X-axis is .... length units. *(Cairo 2012)* (a) 3 (b) – 5 (c) 5 (d) $\sqrt{34}$
7	The distance between the point (4 , –3) and the X-axis = ... length unit. (a) –3 (b) 3 (c) 4 (d) 5
8	The distance between the point (4 , 2) and the y-axis equals .... length unit. (a) 2 (b) 6 (c) 4 (d) 10
9	Distance between point (2 , – 3) and y-axis = ... length units. (a) 2 (b) – 3 (c) $\sqrt{13}$ (d) $\sqrt{5}$
10	The distance between the point (4 , – 3) and y-axis = ... unit length. (a) – 3 (b) 3 (c) 4 (d) 5
11	The distance between the point (3 , 4) and the origin point equals .... (a) 3 (b) 4 (c) 5 (d) 7

12 The distance between the point $\left(\sqrt{5}, -2\right)$ and origin point is .... length unit.

(a) 2 (b) – 2 (c) 3 (d) 8

13 The distance between the point (a , 0) and the point (0 , –1) equals $\sqrt{5}$, then a = ...

(a) 2 (b) –2 (c) ± 2 (d) 5

14 The distance between the two points (a , 0) , (0 , 1) is one length unit , then a = ...

(a) – 1 (b) 0 (c) 1 (d) ± 1

15 A circle its centre is the origin and its radius length is 2 length units , which of the following points belongs to the circle ? *(Alexandria and Assiut 2011)*

(a) (1 , 2) (b) (– 2 , 1) (c) $\left(\sqrt{3}, 1\right)$ (d) $\left(\sqrt{2}, 1\right)$

16 In the Cartesian coordinates plane , the point that is at a distance 2 length units from the origin may be .... *(Cairo 2009)*

(a) (1 , 2) (b) (2 , 1) (c) (0 , 2) (d) (– 3 , 5)

17 If the origin point is a centre of a circle of radius 3 length units , then the point which belonges to the circle is : ....

(a) (1 , 2) (b) (3 , 2) (c) (1 , 3) (d) $\left(-2, \sqrt{5}\right)$

18 The radius length of the circle of center (7 , 4) passing through the point (3 , 1) equals .... unit length.

(a) 7 (b) 6 (c) 5 (d) 4

19 The midpoint of $\overline{OB}$ where O (0 , 0) and B (– 4 , 2) is the point ....

(a) (– 2 , 1) (b) (2 , – 1) (c) (– 2 , 0) (d) (2 , – 4)

20 If : A (0 , 4) and B (6 , 0) , then the coordinates of midpoint of $\overline{AB}$ are ....

(a) (2 , 3) (b) (3 , 2) (c) (6 , 4) (d) (4 , 6)

21 The midpoint of $\overline{AB}$ where A (0 , 6) , B (4 , 0) is ....

(a) (4 , 6) (b) (6 , 4) (c) (2 , 3) (d) (3 , 2)

22 The midpoint of $\overline{AB}$ where A (0 , 8) , B (6 , 0) is the point = ...

(a) (6 , 8) (b) (8 , 6) (c) (3 , 4) (d) (4 , 3)

23 If A (1 , 3) and B (3 , –5) , then the midpoint of $\overline{AB}$ is .... *(Cairo 2011)*

(a) (2 , 1) (b) (2 , 4) (c) (2 , –1) (d) (–2 , 1)

24 If : A (2 , – 2) and B (– 2 , 2) , then the midpoint of $\overline{AB}$ is ....

(a) (– 1 , 1) (b) (1 , – 1) (c) (4 , – 4) (d) (0 , 0)

25 If : A (2 , 4) , B (6 , 0) , then the coordinates of midpoint of $\overline{AB}$ = ...

(a) (4 , 4) (b) (8 , 4) (c) (– 2 , 2) (d) (4 , 2)

26 The midpoint of $\overline{AB}$ where A (2 , 5) , B (4 , 3) is ....

(a) (4 , 5) (b) (5 , 4) (c) (3 , 4) (d) (4 , 3)

27 If : A (3 , – 4) , B (5 , – 2) and C is the midpoint of $\overline{AB}$, then : C = ...

(a) (8 , – 6) (b) (1 , 1) (c) (– 1 , – 1) (d) (4 , – 3)

28 If : $\overline{AB}$ is a diameter of a circle , where A (3 , –5) , B (5 , 1) , then the center of the circle is = ...

(a) (4 , –2) (b) (4 , 2) (c) (2 , 2) (d) (8 , –2)

29 If A (7 , –4) and B (–1 , 0) , then the coordinates of the midpoint of $\overline{AB}$ is ....

(a) (–3 , 2) (b) (3 , 2) (c) (3 , –2) (d) (–3 , –2)

(New Valley 2011)

30 If : C (0 , 4) is the midpoint of $\overline{AB}$ where A (– 1 , – 1) and B (x , y) , then the point B (x , y) = ...

(a) (1 , 9) (b) (– 1 , 9) (c) (1 , – 9) (d) (– 1 , – 9)

31 If C (2 , 1) is the midpoint of $\overline{AB}$ where B (3 , 0) , then A is .... *(6th October 2011)*

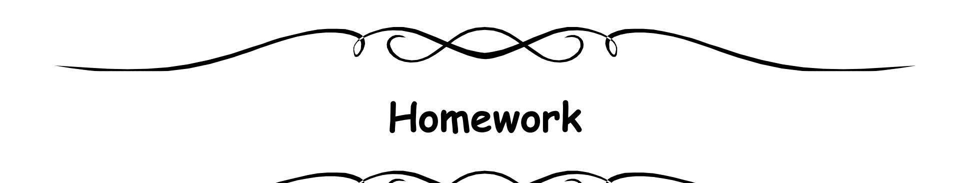
(a) (1 , 2) (b) (2 , 1) (c) (5 , 1) (d) (1 , 5)

32 If : (4 , –3) is the midpoint of $\overline{AB}$ such that A = (3 , –4) , then the coordinates of B is ....

(a) (5 , 2) (b) (5 , – 2) (c) (2 , 5) (d) (0 , 2)

Essay problems:

1	🕮 Prove that the triangle with vertices of points : A (5 , – 5) , B (– 1 , 7) and C (15 , 15) is a right-angled triangle at B , then calculate its area. *(Beni Suef 2013 – El-Monofia 2014)* « 120 square unit »
2	**In each of the following , prove that the points A , B , C and D are vertices of a parallelogram where :** A (–1 , 1) , B (0 , 5) , C (5 , 6) and D (4 , 2) *(Suez 2011)*
3	**Prove that :** The points A (0 , 1) , B (4 , 5) , C (1 , 8) and D (– 3 , 4) are vertices of a rectangle and find its diagonal length. *(Souhag 2009)* « $5\sqrt{2}$ length units»
4	**Prove that :** The points A (3 , – 1) , B (– 4 , 6) and C (2 , – 2) lie on the same circle whose centre is M (– 1 , 2) , then find the circumference of the circle where $\pi = 3.14$ *(Alex. 2015 – Cairo 2015 – El-Sharkia 2013)* « 31.4 length units »
5	🕮 If A (x , 3) , B (3 , 2) and C (5 , 1) and AB = BC , then find the value of x *(El-Beheira 2015 – Port Said 2014)* « 5 or 1 »
6	If C (4 , 6) is the midpoint of $\overline{AB}$ where A (x , 3) and B (6 , y) , then find the value of each of : x and y *(Cairo 2015)* « 2 , 9 »
7	🕮 **If C is the midpoint of $\overline{AB}$, then find x , y in each of the following cases :** A (x , 3) , B (6 , y) , C (4 , 6) *(Luxor 2013)* « 2 , 9 »
8	$\overline{AB}$ is a diameter in a circle M , if B (8 , 11) and M (5 , 7) **Find :** (1) The coordinates of A (2) The perimeter of the circle. where ($\pi = 3.14$) *(Assiut 2014)* « A (2 , 3) , 31.4 length unit »
9	🕮 If the points A (3 , 2) , B (4 , –3) , C (–1 , –2) and D (–2 , 3) are vertices of the rhombus. **Find :** (1) The coordinates of the point of intersection of the two diagonals. (2) The area of the rhombus ABCD *(Suez 2014)* « (1 , 0) , 24 square unit »



Homework

Choose the correct answer :

1	The length of the line segment which is drawn between the points (0 , 0) and (5 , 12) = ... (a) 5 (b) 7 (c) 12 (d) 13
2	The distance between the point (2 , –3) and the x-axis equals .... units length. (a) 2 (b) 3 (c) –3 (d) –2
3	The distance between the point (4 , 2) and the y-axis equals .... length unit. (a) 2 (b) 6 (c) 4 (d) 10
4	✱ The distance between the point (10 , 0) and the origin point equals .... length unit. (a) 3 (b) 6 (c) 8 (d) 10
5	📖 A circle its centre is the origin and its radius length is 2 length units , which of the following points belongs to the circle ? *(Alexandria and Assiut 2011)* (a) (1 , 2) (b) (– 2 , 1) (c) $(\sqrt{3}, 1)$ (d) $(\sqrt{2}, 1)$
6	The length of the line segment which is drawn between the two points (0 , 0) and (6 , 8) = ... length unit. (a) 6 (b) 8 (c) 10 (d) 14
7	The distance between the point (2 , – 5) and the x-axis = ... length unit. (a) 5 (b) 2 (c) – 5 (d) – 3
8	Distance between point (2 , – 3) and y-axis = ... length units. (a) 2 (b) – 3 (c) $\sqrt{13}$ (d) $\sqrt{5}$
9	The distance between the point $(\sqrt{5}, -2)$ and origin point is .... length unit. (a) 2 (b) – 2 (c) 3 (d) 8

10 In the Cartesian coordinates plane , the point that is at a distance 2 length units from the origin may be .... *(Cairo 2009)*

(a) (1 , 2) (b) (2 , 1) (c) (0 , 2) (d) (– 3 , 5)

11 If : C (– 6 , 0) , D (0 , 8) , then CD = ... length unit.

(a) 14 (b) 10 (c) 5 (d) 2

12 The distance between the point (3 , – 5) and X-axis is .... length units. *(Cairo 2012)*

(a) 3 (b) – 5 (c) 5 (d) $\sqrt{34}$

13 The distance between the point (4 , – 3) and y-axis = ... unit length.

(a) – 3 (b) 3 (c) 4 (d) 5

14 The distance between the point (a , 0) and the point (0 , –1) equals $\sqrt{5}$, then a = ...

(a) 2 (b) –2 (c) ± 2 (d) 5

15 If the origin point is a centre of a circle of radius 3 length units , then the point which belonges to the circle is : ....

(a) (1 , 2) (b) (3 , 2) (c) (1 , 3) (d) $\left(-2 , \sqrt{5}\right)$

16 The length of the line segment that drawn between the two points (3 , 2) and (–1 , 5) = ... length unit.

(a) 15 (b) 3 (c) 5 (d) 10

17 The distance between the point (4 , –3) and the X-axis = ... length unit.

(a) –3 (b) 3 (c) 4 (d) 5

18 The distance between the point (3 , 4) and the origin point equals ....

(a) 3 (b) 4 (c) 5 (d) 7

19 The distance between the two points (a , 0) , (0 , 1) is one length unit , then a = ...

(a) – 1 (b) 0 (c) 1 (d) ± 1

20 The radius length of the circle of center (7 , 4) passing through the point (3 , 1) equals .... unit length.

(a) 7 (b) 6 (c) 5 (d) 4

21 The midpoint of $\overline{OB}$ where O (0 , 0) and B (– 4 , 2) is the point ....

(a) (– 2 , 1) (b) (2 , – 1) (c) (– 2 , 0) (d) (2 , – 4)

22 If : A (0 , 4) and B (6 , 0) , then the coordinates of midpoint of $\overline{AB}$ are ....

(a) (2 , 3) (b) (3 , 2) (c) (6 , 4) (d) (4 , 6)

23 The midpoint of $\overline{AB}$ where A (0 , 6) , B (4 , 0) is ....

(a) (4 , 6) (b) (6 , 4) (c) (2 , 3) (d) (3 , 2)

24 The midpoint of $\overline{AB}$ where A (0 , 8) , B (6 , 0) is the point = ...

(a) (6 , 8) (b) (8 , 6) (c) (3 , 4) (d) (4 , 3)

25 If A (1 , 3) and B (3 , –5) , then the midpoint of $\overline{AB}$ is .... *(Cairo 2011)*

(a) (2 , 1) (b) (2 , 4) (c) (2 , –1) (d) (–2 , 1)

26 If : A (2 , – 2) and B (– 2 , 2) , then the midpoint of $\overline{AB}$ is ....

(a) (– 1 , 1) (b) (1 , – 1) (c) (4 , – 4) (d) (0 , 0)

27 If : A (2 , 4) , B (6 , 0) , then the coordinates of midpoint of $\overline{AB}$ = ...

(a) (4 , 4) (b) (8 , 4) (c) (– 2 , 2) (d) (4 , 2)

28 The midpoint of $\overline{AB}$ where A (2 , 5) , B (4 , 3) is ....

(a) (4 , 5) (b) (5 , 4) (c) (3 , 4) (d) (4 , 3)

29 If : A (3 , – 4) , B (5 , – 2) and C is the midpoint of $\overline{AB}$, then : C = ...

(a) (8 , – 6) (b) (1 , 1) (c) (– 1 , – 1) (d) (4 , – 3)

30 If : $\overline{AB}$ is a diameter of a circle , where A (3 , –5) , B (5 , 1) , then the center of the circle is = ...

(a) (4 , –2) (b) (4 , 2) (c) (2 , 2) (d) (8 , –2)

31 If A (7 , –4) and B (–1 , 0) , then the coordinates of the midpoint of $\overline{AB}$ is ....

(a) (–3 , 2) (b) (3 , 2) (c) (3 , –2) (d) (–3 , –2)

(New Valley 2011)

32 If : C (0 , 4) is the midpoint of $\overline{AB}$ where A (– 1 , – 1) and B (x , y) , then the point B (x , y) = ...

(a) (1 , 9) (b) (– 1 , 9) (c) (1 , – 9) (d) (– 1 , – 9)

33 If C (2 , 1) is the midpoint of $\overline{AB}$ where B (3 , 0) , then A is .... *(6th October 2011)*

(a) (1 , 2) (b) (2 , 1) (c) (5 , 1) (d) (1 , 5)

34 If : (4 , –3) is the midpoint of $\overline{AB}$ such that A = (3 , –4) , then the coordinates of B is ....

(a) (5 , 2) (b) (5 , – 2) (c) (2 , 5) (d) (0 , 2)

35 ✱ If the point of the origin O (0 , 0) is the midpoint of the line segment $\overline{AB}$ where A (5 , – 2) , then the coordinates of the point B is ....

(a) (– 5 , 2) (b) (5 , – 2) (c) (– 2 , 5) (d) (0 , 5)

Essay problems:

1 🕮 Prove that the triangle with vertices of points : A (5 , – 5) , B (– 1 , 7) and C (15 , 15) is a right-angled triangle at B , then calculate its area.

(Beni Suef 2013 – El-Monofia 2014) « 120 square unit »

2 If A (–1 , –1) , B (2 , 3) and C (6 , 0)

Prove that : Δ ABC is a right-angled triangle , then find its area.

(Alexandria – Beni Suef 2011) « 12.5 square units »

3 **In each of the following , prove that the points A , B , C and D are vertices of a parallelogram where :**

A (–1 , 1) , B (0 , 5) , C (5 , 6) and D (4 , 2) *(Suez 2011)*

4 **In each of the following , prove that the points A , B , C and D are vertices of a parallelogram where :**

A (– 2 , 4) , B (5 , – 3) , C (7 , 1) and D (0 , 8) *(Souhag 2008)*

5 **Prove that :** The points A (0 , 1) , B (4 , 5) , C (1 , 8) and D (– 3 , 4) are vertices of a rectangle and find its diagonal length. *(Souhag 2009)* « $5\sqrt{2}$ length units»

6 **Prove that :** The points A (3 , 3) , B (0 , 3) , C (0 , 0) and D (3 , 0) in the Cartesian coordinates plane are vertices of a square and calculate the length of its diagonal and its area. *(Luxor 2009)* « $3\sqrt{2}$ length units , 9 square units»

7	**Prove that :** The points A $(3, -1)$, B $(-4, 6)$ and C $(2, -2)$ lie on the same circle whose centre is M $(-1, 2)$, then find the circumference of the circle where $\pi = 3.14$ *(Alex. 2015 – Cairo 2015 – El-Sharkia 2013)* « 31.4 length units »
8	If A $(2, x)$ and B $(3, -1)$, AB $= \sqrt{17}$ length units, then find : x *(El-Dakahlia 2013)* « 3 or – 5 »
9	**Find the value of a in each of the following cases :** If the distance between the two points $(a, 7)$, $(-2, 3)$ equals 5 length unit. *(Luxor 2013)* « 1 or – 5 »
10	If A $(x, 3)$, B $(3, 2)$ and C $(5, 1)$ and AB = BC, then find the value of x *(El-Beheira 2015 – Port Said 2014)* « 5 or 1 »
11	If C $(6, -4)$ is the midpoint of $\overline{AB}$ where A $(5, -3)$ Find the coordinates of the point B *(Beni Suef 2014 – El-Beheira 2013)* « $(7, -5)$ »
12	Find the value of each of a and b that satisfies that $(2a - 3, a - b)$ is the midpoint of the line segment whose terminals $(7, -1)$ and $(3, 7)$ *(EL-Fayoum 2012)* « 4, 1 »
13	$\overline{AB}$ is a diameter in a circle M, if B $(8, 11)$ and M $(5, 7)$ **Find :** (1) The coordinates of A (2) The perimeter of the circle. where $(\pi = 3.14)$ *(Assiut 2014)* « A $(2, 3)$, 31.4 length unit »
14	ABCD is a parallelogram where A $(3, 2)$, B $(4, -5)$ and C $(0, -3)$ Find the coordinates of the intersection point of its diagonals, then find the coordinates of the point D *(El-Monofia 2015 – Giza 2015)* « $\left(1\frac{1}{2}, -\frac{1}{2}\right)$, $(-1, 4)$ »
15	**Prove that :** The points A $(6, 0)$, B $(2, -4)$ and C $(-4, 2)$ are the vertices of a right-angled triangle at B, then find the coordinates of D that make the figure ABCD a rectangle. *(Kafr El-Sheikh 2014 – Assiut 2011)* « D $(0, 6)$ »

Sheet (13)

The Slope of the Straight Line

Sheet (13)
The Slope of the Straight Line

Prelude

You studied before the slope of the straight line given two points on it.

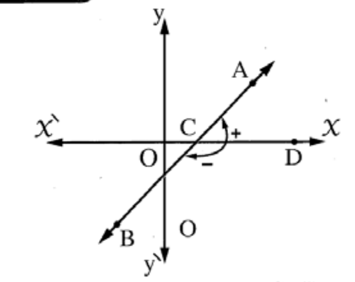
If A and B are two points in the coordinates plane where A (x_1, y_1) and B (x_2, y_2) , then :

$$\text{The slope of the straight line } \overleftrightarrow{AB} = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{where } x_1 \neq x_2$$

The positive measure and the negative measure of an angle

In the opposite figure :

If $\overleftrightarrow{AB}$ intersects the x-axis at the point C , then $\overleftrightarrow{AB}$ makes two angles with the positive direction of the x-axis.



The slope of the straight line

Definition

The slope of the straight line is the tangent of the positive angle which this straight line makes with the positive direction of the x-axis.

i.e. The slope of the straight line = tan θ where θ is the measure of the positive angle which the straight line makes with the positive direction of the x-axis.

Notice that

The straight line passes through the two points (2 , 0) and (7 , 5) , then :

the slope of the straight line $L = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 0}{7 - 2} = \frac{5}{5} = 1$

Remark

The angle which the straight line L makes with the positive direction of the x-axis takes one of the following cases :

1 Acute angle	**2** Obtuse angle	**3** Zero angle	**4** Right angle
The slope is positive	The slope is negative	The slope is zero	The slope is undefined

The relation between the two slopes of the two parallel straight lines

Also, we can deduce the opposite :

If $m_1 = m_2$, then $L_1 // L_2$

i.e. If the two straight lines have equal slopes, then the two straight lines are parallel.

The relation between the slopes of the two perpendicular (orthogonal) straight lines

If L_1 and L_2 are two straight lines of slopes m_1 and m_2 respectively and $L_1 \perp L_2$, then $m_1 \times m_2 = -1$, unless one of them is parallel to one of the coordinate axes.

i.e. The product of the slopes of the perpendicular straight lines $= -1$

and vice versa :

Remark

If $L_1 \perp L_2$, the slope of L_1 is m_1 and the slope of L_2 is m_2, then $m_2 = \frac{-1}{m_1}$, $m_1 = \frac{-1}{m_2}$

For example :

- If the slope of the straight line L is 2, then the slope of the perpendicular to it $= -\frac{1}{2}$
- If the slope of the straight line L is $-\frac{2}{3}$, then the slope of the perpendicular to it $= \frac{3}{2}$

Remarks to solve the problems on quadrilateral

- **To prove that a quadrilateral is a trapezium, we prove that :**
  Two opposite sides are parallel and the other two sides are not parallel.
- **To prove that a quadrilateral is a parallelogram, we prove only one of the following properties :**
  1. Each two opposite sides are parallel.
  2. Each two opposite sides are equal in length.
  3. Two opposite sides are parallel and equal in length.
  4. The two diagonals bisect each other.
- **To prove that a quadrilateral is a rectangle, rhombus or square, we prove at first that the quadrilateral is a parallelogram, then :**

– To prove that the parallelogram is a rectangle, we prove only one of the following two properties :

1. Two adjacent sides are perpendicular.
2. The two diagonals are equal in length.

– To prove that the parallelogram is a rhombus, we prove only one of the following two properties :

1. Two adjacent sides are equal in length.
2. The two diagonals are perpendicular.

– To prove that the parallelogram is a square, we prove only one of the following properties :

1. Two adjacent sides are perpendicular and equal in length.
2. Two adjacent sides are perpendicular and its diagonals are perpendicular.
3. Two diagonals are equal in length and perpendicular.
4. Two adjacent sides are equal in length and its two diagonals are equal in length.

Choose the correct answer :

1	The slope of straight line which parallel to the X-axis is .... (a) 1 (b) – 1 (c) 0 (d) unknown
2	The slope of a straight line which makes an angle of measure 45° with the positive direction of X-axis = ... (a) 1 (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{3}$ (d) otherwise.
3	The slope of straight line which is parallel to the straight line passing through the two points (2 , 3) , (– 2 , 1) equals .... (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) – 4 (d) – 2
4	If the slope of a straight line more than zero , then the type of the positive angle which it makes with the positive direction of X-axis is .... *(Damietta 2011)* (a) zero. (b) acute. (c) right. (d) obtuse.
5	If : m_1 is the slope of the line L_1 and m_2 is the slope of the line L_2 and $L_1 // L_2$, then .... (a) $m_1 m_2 = 1$ (b) $m_1 m_2 = -1$ (c) $m_1 = -m_2$ (d) $m_1 = m_2$
6	If m_1 and m_2 are two slopes of two straight lines L_1 and L_2 respectively and $m_1 - m_2 = 0$, then .... (a) $L_1 \perp L_2$ (b) $L_1 // L_2$ (c) L_1 and L_2 are intersecting. (d) otherwise.
7	If m_1 and m_2 are two slopes of two perpendicular straight lines, then .... *(Qena 2012)* (a) $m_1 = m_2$ (b) $m_1 = -m_2$ (c) $m_1 m_2 = -1$ (d) $m_1 m_2 = 1$
8	If : m_1 , m_2 are the slopes of two perpendicular straight lines, then $m_1 \times m_2 =$ .... (a) 1 (b) $\frac{1}{2}$ (c) – 1 (d) – 2

9	The two straight lines whose slopes are $\frac{3}{5}$ and $\frac{-5}{3}$ are ……… *(Fayoum 2012)* (a) parallel. (b) perpendicular. (c) coincident. (d) not perpendicular.
10	Two parallel straight lines of slopes m_1 and m_2 If : $m_1 = \frac{-1}{3}$, then $m_2 =$ …………… (a) $\frac{1}{3}$ (b) 3 (c) – 3 (d) $-\frac{1}{3}$
11	If : m_1 , m_2 are the slopes of two perpendicular straight lines , $m_1 = 0.75$, then $m_2 =$ …………… (a) $-\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $-\frac{4}{3}$ (d) $\frac{3}{4}$
12	If : $\frac{-2}{3}$, $\frac{k}{2}$ are the slopes of two parallel straight lines , then k = …………… (a) $\frac{-4}{3}$ (b) $\frac{-3}{4}$ (c) $\frac{1}{3}$ (d) 3
13	If : $\frac{k}{3}$, $\frac{-2}{7}$ are the slopes of two parallel straight lines , then k = …………… (a) $\frac{-6}{7}$ (b) $\frac{-7}{6}$ (c) $\frac{2}{7}$ (d) 7
14	If : $\frac{-2}{3}$, $\frac{k}{6}$ are the slopes of two parallel straight lines , then : k = …………… (a) – 12 (b) – 4 (c) – 9 (d) 4
15	If : $\overleftrightarrow{AB} // \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{2}{3}$, then the slope of $\overleftrightarrow{CD} =$ …………… (a) $\frac{-2}{3}$ (b) 5 (c) – 6 (d) $\frac{2}{3}$
16	If : $\overleftrightarrow{AB} // \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = 4$, then the slope of $\overleftrightarrow{CD} =$ …………… (a) – 1 (b) $\frac{1}{4}$ (c) $-\frac{1}{4}$ (d) 4
17	If $\overleftrightarrow{AB} // \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{-3}{2}$, then the slope of $\overleftrightarrow{CD} =$ …………… (a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{-2}{3}$ (d) $\frac{-3}{2}$
18	If $\overline{AB} // \overline{CD}$ and the slope of $\overline{AB} = 0.75$, then the slope of $\overline{CD} =$ …………… (a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) 0.25 (d) 0.57

19 * If $\overleftrightarrow{AB}$ parallels the X-axis where A (2 , – 5) , B (4 , k) , then : k = ...

(a) $\frac{2}{5}$ (b) $\frac{-2}{5}$ (c) 5 (d) – 5

20 If the straight line $\overleftrightarrow{AB}$ is parallel to X-axis , where A (8 , 3) , B (2 , k) , then k = ...

(a) 3 (b) 8 (c) 2 (d) zero

21 If the straight line $\overleftrightarrow{AB}$ is parallel to the X-axis where A (5 , – 3) and B (4 , k) , then k = ...

(a) – 3 (b) 5 (c) 4 (d) 1

22 If : $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{1}{2}$, then the slope of $\overleftrightarrow{CD}$ = ...

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 2 (d) – 2

23 If : $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = -\frac{2}{3}$, then the slope of $\overleftrightarrow{CD}$ = ...

(a) $\frac{3}{2}$ (b) $\frac{-3}{2}$ (c) $-\frac{2}{3}$ (d) $\frac{2}{3}$

24 If : $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{2}{3}$, then the slope of $\overleftrightarrow{CD}$ = ...

(a) $\frac{3}{2}$ (b) $\frac{-3}{2}$ (c) $\frac{2}{3}$ (d) $\frac{4}{9}$

25 * The straight line that makes with the positive direction of the X-axis an angle of measure 45° , its slope is ....

(a) 45° (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) $\frac{\sqrt{3}}{2}$

26 The straight line passes through two points (a , 0) and (0 , 4) perpendicular to the one which makes an angle of measure 45° with the positive direction of X-axis , then a = ...

(a) 4 (b) –4 (c) 1 (d) –1

(Monofia 2012)

27 The straight line which passes through the two points (1 , y) , (3 , 4) its slope is tan 45°, then y = ...

(a) 1 (b) 2 (c) – 1 (d) 4

Essay problems:

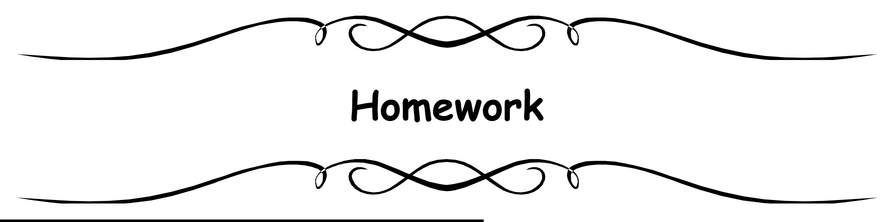
1 Find the slope of the straight line which is perpendicular to the straight line which passes through the two points A (2 , – 3) , B (3 , 5) *(Matrouh 2009)* « $-\frac{1}{8}$ »

2 **Prove that :** The straight line which passes through the two points $(4 , 3\sqrt{3})$ and $(5 , 2\sqrt{3})$ is perpendicular to the straight line which makes an angle of measure 30° with the positive direction of X-axis. *(El-Beheira 2013)*

3 **Prove that :** The points A (1 , 1) , B (2 , 3) and C (0 , – 1) are collinear. *(Cairo 2013)*

4 The triangle whose vertices are A (3 , –1) , B (X , 3) and C (5 , 3) is a right-angled triangle at A , find the value of X *(Cairo 2011)* « –5 »

5 📖 **Prove that :** The points A (– 1 , 1) , B (0 , 5) , C (4 , 2) and D (5 , 6) are the vertices of the parallelogram ABDC *(Luxor 2012)*



Homework

Choose the correct answer :

1 The slope of straight line which parallel to the X-axis is ....

(a) 1 (b) – 1 (c) 0 (d) unknown

2 The slope of a straight line which makes an angle of measure 45° with the positive direction of X-axis = ...

(a) 1 (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{3}$ (d) otherwise.

3 The slope of straight line which is parallel to the straight line passing through the two points (2 , 3) , (– 2 , 1) equals ....

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) – 4 (d) – 2

4 If the slope of a straight line more than zero , then the type of the positive angle which it makes with the positive direction of X-axis is .... *(Damietta 2011)*

(a) zero. (b) acute. (c) right. (d) obtuse.

5	If : m_1 is the slope of the line L_1 and m_2 is the slope of the line L_2 and $L_1 // L_2$, then .... (a) $m_1 m_2 = 1$ (b) $m_1 m_2 = -1$ (c) $m_1 = -m_2$ (d) $m_1 = m_2$
6	If m_1 and m_2 are two slopes of two straight lines L_1 and L_2 respectively and $m_1 - m_2 = 0$, then .... (a) $L_1 \perp L_2$ (b) $L_1 // L_2$ (c) L_1 and L_2 are intersecting. (d) otherwise.
7	If m_1 and m_2 are two slopes of two perpendicular straight lines, then .... *(Qena 2012)* (a) $m_1 = m_2$ (b) $m_1 = -m_2$ (c) $m_1 m_2 = -1$ (d) $m_1 m_2 = 1$
8	If : m_1 , m_2 are the slopes of two perpendicular straight lines, then $m_1 \times m_2 =$ .... (a) 1 (b) $\frac{1}{2}$ (c) -1 (d) -2
9	The two straight lines whose slopes are $\frac{3}{5}$ and $\frac{-5}{3}$ are .... *(Fayoum 2012)* (a) parallel. (b) perpendicular. (c) coincident. (d) not perpendicular.
10	Two parallel straight lines of slopes m_1 and m_2 If : $m_1 = \frac{-1}{3}$, then $m_2 =$ .... (a) $\frac{1}{3}$ (b) 3 (c) -3 (d) $-\frac{1}{3}$
11	If : m_1 , m_2 are the slopes of two perpendicular straight lines , $m_1 = 0.75$, then $m_2 =$ .... (a) $-\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $-\frac{4}{3}$ (d) $\frac{3}{4}$
12	If : $\frac{-2}{3}$, $\frac{k}{2}$ are the slopes of two parallel straight lines , then k = ... (a) $\frac{-4}{3}$ (b) $\frac{-3}{4}$ (c) $\frac{1}{3}$ (d) 3
13	If : $\frac{k}{3}$, $\frac{-2}{7}$ are the slopes of two parallel straight lines , then k = ... (a) $\frac{-6}{7}$ (b) $\frac{-7}{6}$ (c) $\frac{2}{7}$ (d) 7
14	If : $\frac{-2}{3}$, $\frac{k}{6}$ are the slopes of two parallel straight lines , then : k = ... (a) -12 (b) -4 (c) -9 (d) 4

15

If : $\overleftrightarrow{AB} \,//\, \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{2}{3}$, then the slope of $\overleftrightarrow{CD} =$ ....

(a) $\frac{-2}{3}$ (b) 5 (c) – 6 (d) $\frac{2}{3}$

16

If : $\overleftrightarrow{AB} \,//\, \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = 4$, then the slope of $\overleftrightarrow{CD} =$ ....

(a) – 1 (b) $\frac{1}{4}$ (c) $-\frac{1}{4}$ (d) 4

17

If $\overleftrightarrow{AB} \,//\, \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{-3}{2}$, then the slope of $\overleftrightarrow{CD} =$ ....

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{-2}{3}$ (d) $\frac{-3}{2}$

18

If $\overline{AB} \,//\, \overline{CD}$ and the slope of $\overline{AB} = 0.75$, then the slope of $\overline{CD} =$ ....

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) 0.25 (d) 0.57

19

✱ If $\overleftrightarrow{AB}$ parallels the X-axis where A (2 , – 5) , B (4 , k) , then : k = ...

(a) $\frac{2}{5}$ (b) $\frac{-2}{5}$ (c) 5 (d) – 5

20

If the straight line $\overleftrightarrow{AB}$ is parallel to X-axis , where A (8 , 3) , B (2 , k) , then k = ...

(a) 3 (b) 8 (c) 2 (d) zero

21

If the straight line $\overleftrightarrow{AB}$ is parallel to the X-axis where A (5 , – 3) and B (4 , k) , then k = ...

(a) – 3 (b) 5 (c) 4 (d) 1

22

If : $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{1}{2}$, then the slope of $\overleftrightarrow{CD} =$ ....

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 2 (d) – 2

23

If : $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = -\frac{2}{3}$, then the slope of $\overleftrightarrow{CD} =$ ....

(a) $\frac{3}{2}$ (b) $\frac{-3}{2}$ (c) $-\frac{2}{3}$ (d) $\frac{2}{3}$

24

If : $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{2}{3}$, then the slope of $\overleftrightarrow{CD} =$ ....

(a) $\frac{3}{2}$ (b) $\frac{-3}{2}$ (c) $\frac{2}{3}$ (d) $\frac{4}{9}$

25	✱ The straight line that makes with the positive direction of the $\mathcal{X}$-axis an angle of measure 45° , its slope is .... (a) 45° (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) $\frac{\sqrt{3}}{2}$
26	The straight line passes through two points (a , 0) and (0 , 4) perpendicular to the one which makes an angle of measure 45° with the positive direction of $\mathcal{X}$-axis , then a = ... (a) 4 (b) –4 (c) 1 (d) –1 *(Monofia 2012)*
27	The straight line which passes through the two points (1 , y) , (3 , 4) its slope is tan 45°, then y = ... (a) 1 (b) 2 (c) – 1 (d) 4
28	The length of the line segment drawn from the point (0 , 0) to the point (– 4 , 3) equals .... units of length. (a) 3 (b) 4 (c) $\sqrt{7}$ (d) 5
29	The distance between the two points (5 , 0) and (0 , – 12) = ... length units. (a) 7 (b) 13 (c) 17 (d) 10
30	If : C (– 6 , 0) , D (0 , 8) , then CD = ... length unit. (a) 14 (b) 10 (c) 5 (d) 2
31	The distance between the point (2 , –3) and the $\mathcal{X}$-axis equals .... units length. (a) 2 (b) 3 (c) –3 (d) –2
32	The distance between the point (2 , – 5) and the $\mathcal{X}$-axis = ... length unit. (a) 5 (b) 2 (c) – 5 (d) – 3
33	The distance between the point (3 , – 5) and $\mathcal{X}$-axis is .... length units. *(Cairo 2012)* (a) 3 (b) – 5 (c) 5 (d) $\sqrt{34}$
34	The distance between the point (4 , –3) and the $\mathcal{X}$-axis = ... length unit. (a) –3 (b) 3 (c) 4 (d) 5
35	The distance between the point (4 , 2) and the y-axis equals .... length unit. (a) 2 (b) 6 (c) 4 (d) 10

36	Distance between point (2 , – 3) and y-axis = ... length units. (a) 2 (b) – 3 (c) $\sqrt{13}$ (d) $\sqrt{5}$
37	The distance between the point (4 , – 3) and y-axis = ... unit length. (a) – 3 (b) 3 (c) 4 (d) 5
38	The distance between the point (3 , 4) and the origin point equals .... (a) 3 (b) 4 (c) 5 (d) 7
39	The distance between the point $(\sqrt{5} , -2)$ and origin point is .... length unit. (a) 2 (b) – 2 (c) 3 (d) 8
40	The distance between the point (a , 0) and the point (0 , –1) equals $\sqrt{5}$, then a = ... (a) 2 (b) –2 (c) ± 2 (d) 5

Essay problems:

1 🕮 **Prove that :** The straight line passing through the two points (2 , – 1) and (6 , 3) is parallel to the straight line that makes an angle of measure 45° with the positive direction of the x-axis. *(Kafr El-Sheikh 2011)*

2 🕮 If the straight line L_1 passes through the two points (3 , 1) and (2 , k) and the straight line L_2 makes with the positive direction of the x-axis an angle whose measure is 45° , then find k if the two straight lines L_1 and L_2 are :

(1) parallel (2) perpendicular *(Aswan 2014)* « 0 , 2 »

3 🕮 If the points (0 , 1) , (A , 3) and (2 , 5) are located on one straight line. Then find the value of A *(El-Gharbia 2014)* « 1 »

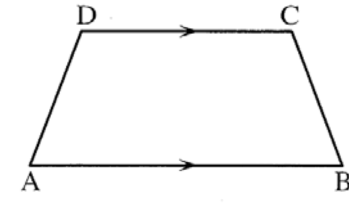
4 🕮 If A (– 1 , – 1) , B (2 , 3) and C (6 , 0) , **prove that :** the triangle ABC is a right-angled triangle at B *(Suez 2014)*

5 🕮 Prove by using the slope that the points A (– 1 , 3) , B (5 , 1) , C (6 , 4) and D (0 , 6) are the vertices of the rectangle ABCD *(Beni Suef 2013)*

6 🕮 **In the drawn figure :**

ABCD is a trapezoid where $\overline{AB} \parallel \overline{CD}$, A (9 , – 2) , B (3 , 2) , C (x , – x) and D (4 , – 3)

Find the coordinates of the point C



(Alex. 2014) « (1 , – 1) »

Sheet (14)

The Equation of the Straight Line Given its Slope and the Intercepted

Sheet (14)
The Equation of the Straight Line Given its Slope and the Intercepted

First Finding the slope of a straight line and the length of the intercepted part from y-axis.

If the equation of a straight line in the form : $y = m\,x + c$, then :

- The slope of the straight line = m
- The length of the intercepted part from y-axis = $|c|$
  and it passes through the point (0 , c)

For Example : -

- The straight line whose equation is $y = \frac{1}{2}x + 7$
  its slope = $\frac{1}{2}$
  and the intercepted part from y-axis = 7 length units and passes through the point (0 , 7)
- The straight line whose equation is $y = 3x - 5$, its slope = 3
  and cuts from the negative side of y-axis a part of 5 length units and passes through the point (0 , – 5)

Remarks

If the equation of a straight line in the form : $a\,x + b\,y + c = 0$

, then the slope of the straight line = $\frac{-\text{coefficient of } x}{\text{coefficient of } y}$

and the straight line cuts y-axis at the point $\left(0 , \frac{-c}{b}\right)$

i.e. The length of the intercepted part from y-axis = $\left|\frac{-c}{b}\right|$

For Example : -

1 The straight line whose equation : $x - 2y + 3 = 0$

Its slope = $\frac{-1}{-2} = \frac{1}{2}$ and cut y-axis at the point $\left(0 , \frac{3}{2}\right)$

i.e. The straight line intercepts a part of length equals $\frac{3}{2}$ length unit from the positive side of y-axis.

2 The straight line whose equation : $3x + y + 4 = 0$

Its slope = – 3 and cut y-axis at the point (0 , – 4)

i.e. The straight line intercepts a part of length equals 4 length units from the negative side of y-axis.

Second Finding the equation of the straight line given its slope and the length of intercepted part of y-axis

The straight line whose slope = m and cuts y-axis at the point (0 , c) its equation is in the form :

$y = mx + c$

1 The equation of the straight line which passes through the origin point O (0 , 0) is $y = mx$, where m is the slope of the straight line.

2 The equation of x-axis is $y = 0$

3 The equation of y-axis is $x = 0$

4 The equation of the straight line parallel to x-axis and passes through the point $(0 , \ell)$ is $y = \ell$

5 The equation of the straight line which is parallel to y-axis and passes through the point (k , 0) is $x = k$

Choose the correct answer :

1 The straight line whose equation is $2x - y + 4 = 0$ intercepts a part from y-axis of length .... units.

(a) – 4 (b) 2 (c) – 1 (d) 4

2 The straight line whose equation is $y = 2x - 6$ intercepts from the y-axis a part of length .... unit.

(a) – 6 (b) – 3 (c) 2 (d) 6

3 The straight line whose equation is $y = \frac{2}{3}x + 2$ intercepts from the y-axis a part of length .... length unit.

(a) – 6 (b) – 2 (c) $\frac{2}{3}$ (d) 2

4	The straight line whose equation is : $5y = 2x + 10$ intercepts a part from y-axis , the length of intercepted part is = ... length unit. (a) 10 (b) 5 (c) 2 (d) $2\frac{1}{2}$
5	The straight line whose equation is : $y - 3x = 6$ intercepts from the y-axis a part of length .... units. (a) 6 (b) 2 (c) 3 (d) $-\frac{1}{3}$
6	✱ The straight line : $4y = 3x + 16$ intercepts from the y-axis a part of length .... length units. (a) 3 (b) 4 (c) 5 (d) 16
7	The length of the intercept part of y-axis by the straight line $y = 5x - 4$ equal .... length unit. (a) 1 (b) 5 (c) 4 (d) 9
8	The straight line whose equation is $y = x$ passing through the point .... (a) (1 , 0) (b) (1 , 2) (c) (0 , 0) (d) (0 , 1)
9	The equation of the straight line that passes through the point (1 , 3) and paralled to x-axis is .... (a) $x = 1$ (b) $x = 3$ (c) $y = 1$ (d) $y = 3$
10	The equation of the straight line which passes through the point (2 , – 3) and parallel to x-axis is .... (a) $x = -2$ (b) $y = -3$ (c) $x = 2$ (d) $y = 3$
11	The equation of the straight line which passes through (– 7 , 2) parallel to y-axis is .... (a) $x = 2$ (b) $x = -2$ (c) $y = 7$ (d) $x = -7$
12	The equation of the straight line passing through the origin point and makes an angle of measure 45° with the positive direction of x-axis is .... (a) $x = 1$ (b) $y = 1$ (c) $y = x$ (d) $y = -x$
13	The equation of the straight line when its slope equals 5 and intersects a positive part from the y-axis that equals 7 units is .... (a) $y = 5x - 7$ (b) $y = 7x + 5$ (c) $y = 5x + 7$ (d) $y = 7x - 5$

No.	Question
14	The equation of the straight line whose slope is 1 , passes through the origin point is .... (a) $x = 1$ (b) $y = 1$ (c) $y = x$ (d) $y = -x$
15	The equation of the straight line whose slope = 2 and passes through the origin point is .... (a) $x = 2$ (b) $y = 2$ (c) $y = 2x$ (d) $y = \frac{1}{2}x$
16	The slope of the straight line whose equation : $cx + ay + b = 0$ is .... (a) $\frac{-a}{b}$ (b) $\frac{-a}{c}$ (c) $\frac{-b}{c}$ (d) $\frac{-c}{a}$
17	The slope of the straight line parallel to the straight line $y + 2 = 0$ equals .... (a) -1 (b) 0 (c) 1 (d) undefined.
18	If the straight line L is perpendicular to the straight line whose equation : $y - 2x = 7$, then the slope of L equals .... (a) 3 (b) 2 (c) $-\frac{1}{3}$ (d) $-\frac{1}{2}$
19	The slope of straight line whose equation is : $y = 5 - 3x$ is .... (a) 5 (b) -3 (c) $\frac{5}{3}$ (d) $\frac{3}{5}$
20	The slope of the straight line whose equation : $2x - 3y + 5 = 0$ .... (a) $\frac{-3}{2}$ (b) $\frac{-2}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$
21	The slope of the straight line : $2y = 6x + 1$ is .... (a) $\frac{1}{3}$ (b) 3 (c) -3 (d) $-\frac{1}{3}$
22	The slope of straight line which is perpendicular to straight line : $2x + 3y = 1$ is .... (a) $\frac{2}{3}$ (b) $\frac{-2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{-3}{2}$
23	The straight line whose equation is : $3x - 3y + 5 = 0$ makes a positive angle with the positive direction of x-axis , its measure = ... *(El-Monofia 2011)* (a) 30° (b) 45° (c) 60° (d) 90°
24	If the the straight line $y = x \sin 30° + c$ passing through point (4 , 6) , then c = ... (a) 5 (b) -4 (c) 4 (d) 6

25	If the straight line $a X - 4y = 1$ its slope equals $\frac{1}{2}$, then a = ... (a) – 8 (b) – 2 (c) 2 (d) 8
26	If the straight line whose equation : $X + 3y - 6 = 0$ is perpendicular to the straight line whose equation : $a X - 3y + 7 = 0$, then : a = ... (a) 2 (b) 9 (c) 4 (d) 1
27	If the straight line whose equation is : $y = (a - 1) X + 5$ is parallel to the straight line which passes through the two points (1 , 2) and (3 , 8) , then the value of a = ... *(El-Sharkia 2009)* (a) 3 (b) 4 (c) – 4 (d) 7
28	If the two straight lines : $X + y = 5$ and $k X + 2y = 8$ are both parallel , then k = ... (a) – 2 (b) – 1 (c) 1 (d) 2
29	📖 If the two straight lines : $3X - 4y - 3 = 0$ and $ky + 4X - 8 = 0$ are perpendicular , then k = ... *(El-Fayoum 2011)* (a) – 4 (b) – 3 (c) 3 (d) 4
30	The two straight lines : $y = aX + b$ and $y = cX + d$ are perpendicular , then .... = – 1 *(El-Gharbia 2008)* (a) $a \times d$ (b) $b \times c$ (c) $a \times c$ (d) $b \times d$
31	Area of triangle bounded by straight lines $X = 0$, $y = 0$, $2X + 3y = 6$ equals .... (a) 6 (b) 5 (c) 4 (d) 3
32	📖 The area of the triangle in square units which is bounded by the straight lines $3X - 4y = 12$, $X = 0$, $y = 0$ equals .... *(El-Sharkia 2012)* (a) 6 (b) 7 (c) 12 (d) – 6

Essay problems:

1	**Find the equation of the straight line if :** 📖 Its slope = 2 and intercepts from the positive part of y-axis 7 units. *(Suez 2015)*
2	**Find the equation of the straight line :** Which cuts a part of length 3 units from the negative part of y-axis and is parallel to the line whose equation : $2\,x - 3\,y = 6$ *(El-Beheira 2011)*
3	Which passes through the point $(2\ ,-1)$ and its slope equals 2 *(El-Kalyoubia 2011)*
4	Passing through the point $(-2\ ,3)$ and perpendicular to the straight line whose equation : $y = \frac{1}{2}x - 5$ *(El-Dakahlia 2013)*
5	📖 Passing through the point $(3\ ,-5)$ and it is parallel to the straight line : $x + 2\,y - 7 = 0$ *(Alexandria 2015)*
6	Which passes through the point $(3\ ,2)$ and parallel to the straight line passing through the two points $(5\ ,6)$ and $(-1\ ,2)$ *(Helwan 2009)*
7	Passing through the point $(1\ ,2)$ and perpendicular to the straight line passing through the two points A $(2\ ,-3)$ and B $(5\ ,-4)$ *(Red Sea 2013 – El-Gharbia 2014)*
8	Passing through the point $(2\ ,-2)$ and perpendicular to the straight line which makes an angle of measure 45° with the positive direction of x-axis *(Luxor 2011)*
9	📖 Which passes through the two points $(2\ ,-1)$ and $(1\ ,1)$ *(El-Gharbia 2013)*
10	📖 Which passes through the two points $(4\ ,2)$ and $(-2\ ,-1)$ then prove that it passes through the origin point. *(Suez 2015 – Dakahlia 2012)*
11	Which passes through the midpoint of the line segment $\overline{AB}$ where A $(3\ ,6)$ and B $(-1\ ,4)$ and perpendicular to the straight line whose equation is $2\,y - 4\,x + 11 = 0$ *(Cairo 2009)*
12	**Prove that :** The straight line $\overleftrightarrow{AB}$ is parallel to the straight line whose equation : $x - 2\,y + 8 = 0$ where A $(2\ ,3)$ and B $(-2\ ,1)$ *(El-Fayoum 2011)*

13

Prove that : The straight line whose equation : $2x + y + 8 = 0$ is perpendicular to the straight line passing through A (2 , 3) and B (– 2 , 1) *(Aswan 2012)*

14

If the straight line whose equation : $2x - 3y - 6 = 0$ cuts the x-axis at point A and the y-axis at point B , find : *(El-Sharkia 2013)*

(1) The coordinates of two points A and B

(2) The equation of the straight line passing through the midpoint of $\overline{AB}$ and parallel to the y-axis.

15

If the straight line whose equation : $a x + 2y - 3 = 0$ is parallel to the straight line which passes through the two point (2 , 3) , (1 , 5) which lie on the same plane , then find the value of a *(Souhag 2013) « 4 »*

16

Find the equation of the axis of symmetry of $\overline{XY}$, where X (3 , – 2) and Y (– 5 , 6) *(El-Dakahlia 2012 – Port Said 2014)*

17

Find the equation of the straight line which intercepts from the positive parts of the coordinate axes «x-axis and y-axis» two parts of lengths 4 and 9 length unit respectively. *(Assiut 2012)*

18

The opposite table represents a linear relation :

(1) Find the equation of the straight line.

(2) Find the length of the intercepted part from y-axis.

(3) Find the value of a

x	1	2	3
$y = f(x)$	1	3	a

(Alexandria 2015 – El-Kalyoubia 2013)

19

In the opposite figure :

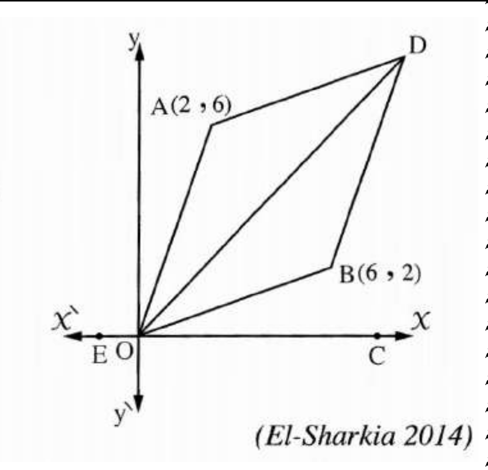
The points A (2 , 6) , O (0 , 0) , B (6 , 2) and D are vertices of the rhombus.

Find :

(1) The coordinates of the point D

(2) The equation of the straight line $\overleftrightarrow{OD}$

(3) m (∠ DOE)



(El-Sharkia 2014)